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GUILHERME RAMALHO GOMEZ

A BIVARIATE RETURN PERIOD COPULA APPLICATION OF FLOOD PEAKS AND VOLUMES.

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Dissertação apresentada ao Programa de Pós-Graduação em Engenharia Civil da Universidade Federal do Ceará como requisito parcial à obtenção do título de Mestre em Engenharia Civil. Área de concentração: Recursos Hídricos.

Orientador: Prof. José Nilson Bezerra Campos.
Coorientador: Prof^{a}. Ticiana Marinho de Carvalho Studart

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GUILHERME RAMALHO GOMEZ

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Dissertation presented to the Graduate Program in Civil Engineering of the Federal University of Ceará, as a partial requirement to obtain a Master’s degree in Civil Engineering. Concentration area: Water resources.

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The history of Science shows that theories are perishable. With every new truth that is revealed we get a better understanding of Nature and our conceptions and views are modified. (NIKOLA TESLA)

RESUMO

A análise de frequências de cheias para o projeto de infraestruturas hidráulicas geralmente é feita com base nos picos de vazões, utilizando distribuições teóricas de probabilidade da vazão máxima anual durante longos períodos de tempo. No entanto, para melhor compreender os eventos extremos de cheias e os seus impactos associados, é importante considerar a influência de outras variáveis correlatas, tais como volume e duração, seguindo uma metodologia de análise multivariada. Para tal abordagem, a aplicação das Copulas tem sido observada em diversos trabalhos publicados no mundo, e os seus resultados mostram impactos positivos dessa abordagem em relação ao método mais tradicionalmente utilizado. No presente trabalho, foi adotada uma metodologia em um estudo de caso para modelar as características conjuntas das inundações a partir do conceito de Copulas, considerando distribuições de probabilidade para pico, volume e duração das cheias e estimados os períodos de retorno conjuntos para análise da frequência de cheias combinadas. O estudo de caso selecionado foi o do reservatório do Castanhão, localizado no Ceará, que está inserido em uma área de clima semiárido no Nordeste do Brasil. As características dessa região a tornam uma zona de alta complexidade hidrológica e altamente vulnerável a eventos extremos de enchentes. No ano de 2004, o Castanhão, que é uma das barragens com maior capacidade de acúmulo de água para múltiplos fins da América Latina, aumentou para mais de 70% sua capacidade total (6.7 bilhões de metros cúbicos) após menos de dois meses de chuva. Portanto, foram coletados dados históricos dos principais postos fluviométricos a jusante da barragem, localizados nos rios Jaguaribe e Salgado e estudadas as relações entre as variáveis computadas. Com a aplicação da metodologia e distribuição das probabilidades conjuntas foi encontrado um período de retorno de cerca de 1.000 anos para a cheia na estação do rio Jaguaribe, enquanto o período de retorno calculado com a abordagem univariada resultou em um valor duas vezes menor. Os resultados indicam que a análise multivariada pode auxiliar a encontrar excepcionalidades muito maiores nos eventos de cheias, não detectadas na abordagem tradicional dos picos anuais de vazão.

Palavras-chave: Cheias. Análise de Frequência. Copulas. Período de retorno conjunto.

ABSTRACT

Flood frequency analysis for the design of hydraulic infrastructures is usually made based on univariate flood peak approaches, using theoretical probability distribution functions of annual maximum peak flood discharges during several years. However, extreme flood events and their associated impacts may be better understood by considering other correlated random variables, such as volume and duration, in a multivariate analysis. For such approach, the Copulas methodology has been applied in several works worldwide, and its results show positive impacts in the analysis of extreme events compared to the traditional univariate methods. In the present work, a methodology is used in a case study to model the joint characteristics of the flood using Copulas concept considering a set of parametric and non-parametric marginal distributions for peak flow, volume, and duration to model the correlated nature among them mathematically. Joint return periods can be easily estimated from Copulas, which represents an additional benefit, as these joint return periods are essential for the analysis of the flood frequency. The selected case study was the Castanhão Reservoir, located in a semi-arid climate area of the state of Ceará, in the Northeast of Brazil (NEB). This semi-arid region's characteristics make NEB a region of high hydrological complexity and highly vulnerable to extreme flood events. In 2004, the Castanhão Reservoir, which has one of the biggest water storage capacity in Latin America, increased to over 70% of its total capacity (6.7 billion m³) after less than two rainy months. Therefore, historical data were collected from the main river gauge stations that flow to Castanhão, located in Jaguaribe River and Salgado River, and the relationship among them was studied. At Jaguaribe river gauge station, the event's joint return period was estimated with Copulas distribuition to almost 1.000 years, which is almost twice as long as the return period of the univariate approach. These results prove that the multivariate analysis can find much greater exceptionalities in flood events, not detected in the univariate approach.

Keywords: Flood. Frequency analysis; Copulas; Joint return period.

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LIST OF ABBREVIATIONS

AIC	Akaike Information Criterion
AMPD	Annual Maximum Peak Discharges
ANA	Agência Nacional de Águas – National Water Agency
CDF	Cumulative Distribution Functions
CV	Coefficient of Variation
DIFV	Daily Flood Inflow Volume
FD	Flood Duration
FUNCEME	Fundação Cearense de Meteorologia - Ceará Meteorology Foundation
FV	Flood Volume
IFM	Inference Function from Margins
ITCZ	Intertropical Convergence Zone
MLE	Maximum Likelihood Estimation
NEB	Northeast of Brazil
PF	Peak Flow
SST	Sea Surface Temperature
TFV	Total Flood Volume
WRC	U.S. Water Resources Council

SUMMARY

1 INTRODUCTION

Floods are extreme events that induces huge social and economic damage for humankind among all other natural disasters. In order to assess and control these impacts due to extreme flood events it is necessary to set up hydraulic infrastructures, such as dams, channels and ditches (Rizwan *et al.*, 2019).

The design, planning, and management of hydraulic infrastructures require detailed knowledge about flood characteristics. Traditionally, most studies are limited to a univariate assessment of maximum annual peak flows frequency aiming to define the severity of a flood event based on its probability of occurrence. Among many others, Villarini and Smith (2010) studied the maximum flood peak of 572 stations in the United States; Calenda et al. (2005) used discharge peak data from hundreds of years of measurements in Tiber River and tested it to fit into Gumbel and Normal distributions and Młyński et al. (2018) evaluated the applicability of a small basin model annual with peak flows for a specific return period in southern Poland.

However, the univariate scheme of frequency analysis usually offers shortcomings when designing critical hydraulic structures like dams, bridges, and culverts. Extreme events such as floods (and droughts) and their associated impacts may be better understood by considering few correlated random variables, such as flood peak, volume, and duration, since they are understood as multivariate phenomenon (Karmakar e Simonovic, 2008; Shiau, 2006). In fact, those flood characteristics present a dependence structure that can be entirely ignored by the univariate analysis, resulting in an incomplete representation of the phenomenon (Alidoost, Su e Stein, 2019; Pontes Filho *et al.*, 2019; Xu *et al.*, 2015).

Although multivariate analysis models have been extensively applied in other fields such as finance, in recent years, this approach in terms of hydrological events started to gain relevance. In the beginning, bivariate distributions such as bivariate Normal, Exponential, Gamma, and Extreme Value distributions were more common as they were easier to use. These bivariate distributions need the same family to model their marginal distributions; though, there are many cases where the different variables do not follow the same distribution. The application of copulas, otherwise, can help combine several marginal distributions in a multivariate distribution (Papaioannou *et al.*, 2016; Requena, Mediero e Garrote, 2013). Such models offer an efficient way of finding reasonable multivariate estimates for hydrological events with a certain likelihood of occurrence. The estimates thus achieved are used as hydraulic structure design variables.

Based on Sklar’s theorem, the copula function is used to map the most suitable marginal distributions of the variables and hence are referred to as copula. Szolgay et al. (2012) used a joint analysis of maximum discharges and volumes with copulas for estimation of design quantities. Favre et al. (2004) and Gaál et al. (2015) introduced a two-dimensional copula to describe the relationship between flood discharges and volumes. Zhang and Singh (2006) stressed that bivariate copula-based distributions of flood peaks vs. volumes and flood volumes vs. durations provide better results than traditional distributions. The works of Pontes Filho et al. (2020) and Mirabbasi et al. (2012), on the other hand, used copula distributions to characterize droughts, which follow theoretical principles similar to those of flood analysis. De Michele et al. (2005) have studied dam safety using copulas by applying the Gumbel copula to verify that the maximum water level was below the dam's crest level by the generated hydrographs with peak volumes.

The studies on floods involving copula distributions show more comprehensive results when compared to those resulting from the univariate analysis; as they allow to account for essential characteristics of the events that otherwise would be neglected, avoiding that certain tailored risks in the design of hydraulic infrastructures and the forecasting of floods to be ignored.

In the present study, a copula function is used to model the joint characteristics and return periods of peak flow, volume, and duration of floods in Castanhão Dam, located in Jaguaribe River, in Ceará State, Brazil. Castanhão is one of the largest multiple-use reservoirs in the Latin America and the largest located on an intermittent river. The construction of such a large dam in an intermittent river was full of controversy since the beginning; it would force the relocation of thousands of people, relocating the urban center of Jaguaribara and, according to experts, would never fill up. Therefore, the construction of the Dam was completed at the end of 2003 with its hydraulic basin almost empty; and in the beginning of 2004 an anoumalous occurrence of several atmospheric mechanisms simultaneously intensified the persistency of rainfall in Northeast Brazil (NEB), especially in its northern part. Such rainy conditions above than normal over NEB filled Castanhão Dam, and its weirs were opened for the first time.

This study's main objective is to investigate the suitability and evaluate the applicability of the selected copula models for the flood peak–volume-duration relationship along the Jaguaribe River using streamflow data from two gauging stations upstream Castanhão Dam in order to calculate the probability of occurrence of 2004 flood episode and compare with traditional univariate methods.

The study is structured as follows. First, the univariate and the copula distribution methodology and theoretical description are shown. The case study section presents the region and characterizes it from both climatological and hydrological points of view. The data used to illustrate the application of the methodology is also presented in this section. The Results section presents the findings of the study, whereas the Discussion and Conclusions sections discuss the findings and their implications and possible topics of further research.

2 OBJECTIVES

2.1 Main Goal

Study the application of multivariate analysis thought Copulas methodology in a real case of flood events, from the point of view of the peaks and volumes, and evaluate its benefits.

2.2 Specific Goals

- Define the best fitted probability distribution function for each variable presented.
- Analyze the joint return periods of extreme flood events in a real case with Copulas.
- Compare the probability of flood events found with the multivariate methodology versus the univariate approach.

3 METHODS

In this section the specific methodology of frequency analysis and distribution used in this research is presented.

3.1 Univariate Analysis

The need for the study of flood flows emerged in the early twentieth century as a core challenge in hydrology, driven by a dam-building boom which in the U.S. ran from1910 to the 1970s. The objective is usually to determine a flood quantile associated with a particular annual exceedance probability, so this quantile estimation process is referred to as flood frequency analysis (Wright, Yu e England, 2020).

Univariate flood frequency analyses have been carried out widely, aiming at characterizing peak flows (PF), flood volumes (FV), or flood durations (FD). Understanding and predicting such characteristics are key issues for designing several hydraulic infrastructures (Requena, Mediero e Garrote, 2013).

The previous characteristics of the flood are considered random variables. Their marginal distribution functions are assumed to follow specific parametric distribution functions in the conventional univariate frequency analysis method.

The most advisable practice, from a statistical point of view, is to test different families of distributions to obtain the model that best fits each flood characteristic in each region since there is no consensus on a single marginal distribution to be used in univariate analysis applied to PF, FV and FD (Karmakar e Simonovic, 2008).

Many parametric distributions have been used to estimate flood frequencies from observed annual flood discharge series. Guru and Jha (2015) found the Lognormal and Pareto distributions more suitable for their region, and Requena et al. (2013) used Gumbel and Extreme Value distributions to model PF. The general extreme value distribution is also recommended as a base method in the United Kingdom (Hall, 1984). The U. S. Water Resources Council (WRC) issued a series of bulletins recommending the Log-Pearson type-III as a base method for all U. S. federal agencies (Adamowski, 1989).

In this study, the Exponential, Gumbel, Gamma, Logistic, Log-Normal, and Weibull distributions were tested aiming to identify the most suitable distribution for modeling PF, FV, and FD, considering that they are the most common types of functions used by several authors for hydrological extremes (Baidya, Singh e Panda, 2020; Barros *et al.*, 2018; NAGHETTINI e FERNANDES, 2007; Ramachandra Rao *et al.*, 2019). The Akaike Information Criterion (AIC) was used as a goodness-of-fit test, and the parameters of the distribution functions were estimated using the Maximum Likelihood Estimation (MLE). AIC estimates the relative quality of statistical models.

3.2 Bivariate Analysis and Copula Fiting

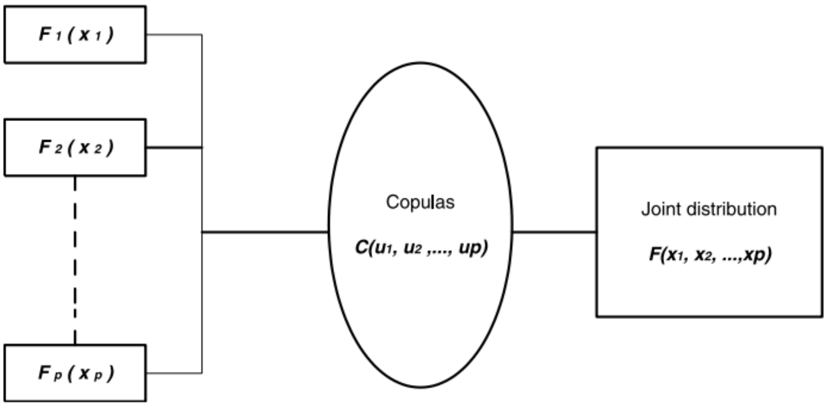
The relationship between PF, FV, and FD is a challenging research topic. Papaioannou et al. (2016), like most other authors, treat the dependence from a purely statistical perspective. However, it would be of interest to understand the hydrological factors controlling the strength of association between peaks and volumes. Many authors considered the Gumbel copula as the copula that best represents the relation between flood peak and flood volume (Zhang e Singh, 2007)

Regarding the bivariate model, this study focused on the use of copula functions to model the dependence structure among marginal distribution functions – Figure 1. The bivariate joint distribution function $H(x, y)$, where "x" and "y" are the random correlated variables, with respective marginal distributions $F_x(x)$ and $F_y(y)$, is given by the copula function $C[F_x(x), F_y(y)]$, according to equation 2:

$$H(x, \mathrm{y}) = C[F_x(x), F_y(y)] = C(u, v) \tag{2}$$

Where $F_x(x)$ and $F_y(y)$ are equal to u and v, with $u, v \in (0,1)$.

Figure 1 - Representation of a copula approach



Source: Adapted from Favre et al. (2004).

The copula functions can be classified in Meta-elliptic and Archimedean copulas: the first is symmetric, presenting no tail dependence; the second is more flexible and can explain upper or lower tail dependence. In this study, three Archimedean copulas, Clayton, Frank, and Gumbel, and two Meta-elliptical copulas, Gaussian and t-Student, were used as candidates to identify the more suitable family to model the dependence structure between the variables.

Archimedean copula family is of major interest to hydrologists on account of its smooth construction and application irrespective of correlation between the hydrological variables (Chen *et al.*, 2010).

Equations 3, 4, 5, 6, and 7 present the candidate copula families' formulations, where $\theta, \rho,$ and v are the copula function parameters.

Clayton $$(\mathrm{u}_1{}^{-\theta} + \mathrm{u}_2{}^{-\theta} - 1)^{\frac{-1}{\theta}}) \tag{3}$$

Frank $$-\frac{1}{\theta}\log\,(1 + \frac{(\mathrm{e}^{-\theta\,\mathrm{u}_1} - 1)(\mathrm{e}^{-\theta\,\mathrm{u}_2} - 1)}{(\mathrm{e}^{-\theta} - 1)}) \tag{4}$$

Gumbel $$\exp\{-[(-\ln \mathrm{u}_1)^{\,\theta} + (-\ln \mathrm{u}_2)^{\,\theta}]^{\frac{1}{\theta}} \tag{5}$$

Gaussian $$\phi_\rho(\phi^{-1}(\mathrm{u}_1), \phi^{-1}(\mathrm{u}_2)) \tag{6}$$

t-Student $$\mathrm{T}_{\rho,\mathrm{v}}(\mathrm{T}_\mathrm{v}^{-1}(\mathrm{u}_1), \mathrm{T}_\mathrm{v}^{-1}(\mathrm{u}_2)) \tag{7}$$

The Inference Function from Margins (IFM) method (Joe, 1997) was used to estimate copula parameters. IFM is a parametric method that consists of the previous definition of marginal distributions used to transform samples in the (0,1) interval. Thus, transformed samples are jointly modeled by estimating the candidate copula's families' parameters using the maximum likelihood method. The minimum value of AIC was used to find the best-fitted model from the candidate copulas. Brechmann and Schepsmeier (2013) defined the AIC relationship with a bivariate copula model and its respective parameter (θ), according to equation 8.

$$AIC = -2\sum_{i=1}^{N} ln[C\ (u_1, v_1|\theta)] + 2k \tag{8}$$

Where i = 1,…, N is the observations of the variables modeled, and k the number of estimated parameters in the model.

One of the great benefits of this approach is that univariate marginal distributions can be defined independently of the joint behavior of the variables involved. Thus, regardless of the family to which the marginal distributions belong, copula functions allow modeling the dependency structure of random variables.

Therefore, joint return periods can be easily estimated from copulas, as will be shown in the next topic, which represents an additional benefit, as these joint return periods are essential for the analysis of the flood frequency.

3.3 Frequency Analysis and Return Periods

To better prepare for upcoming flood events, it is helpful to assign return periods (T) to past events based on the frequency analysis of flood characteristics time series (Haan, 1977) The univariate return period of floods, based on stochastic processes, is derived as follows.

The return period of flood variables (T_X) is described as a function of the expected interarrival time E(L); and the Cumulative Distribution Functions (CDF) of the flood characteristic marginal distributions F_X(x) as defined in Equation (9), where both return periods and E(L) are expressed in years (Shiau, 2006). The E(L) is calculated by adjusting a distribution function to interarrival time and deriving its mean value.

$$\mathrm{T}_X = \frac{E\ (L)}{P\ (X\ \geq x\ )} = \frac{E\ (L)}{1-\ F_X(x)} \tag{9}$$

Due to the multivariate nature of floods, bivariate return periods of hydrological events must be analyzed. Many definitions of multivariate return periods have been provided in the literature depending upon the different functions. The joint flood parameters return periods can be defined in two cases: return period for $X\ \geq x\ or\ Y\ \geq y$ and return period for $X\ \geq x\ and\ Y\ \geq y$, as described by Equations (10) and (11), respectively:

$$\mathrm{T}_{X\ or\ Y} = \frac{E\ (L)}{P\ (X\ \geq x\ or\ Y\ \geq y)} = \frac{E\ (L)}{1-\ F_{XY}(x,y)} = \frac{E\ (L)}{1-C\ (F_X(x),F_Y(y))} \tag{10}$$

$$\mathrm{T}_{X\ \&\ Y} = \frac{E\ (L)}{P\ (X\ \geq x,\ Y\ \geq y)} = \frac{E\ (L)}{1-F_X(x)-\ F_Y(y)+C\ (F_X(x),F_Y(y))} \tag{11}$$

For “or” scenario, one of the flood variable may exceed the arbitrary values but for the “&” scenario, it is required that both the flood variables exceed the recommended level (Michele, De *et al.*, 2005; Rizwan *et al.*, 2019).

4 CASE STUDY

The selected case study was the Castanhão Dam, located in a semi-arid area of Ceará State, in NEB (Figure 2). The characteristics of this semi-arid region make NEB a region of high hydrological complexity, highly vulnerable to extreme events, droughts, or floods (Hastenrath e Heller, 1977; Rebouças, 1997; Silva, Pereira e Almeida, 2012). Under such conditions, river discharges, which are the primary input variable to the artificial reservoirs, have significant year-to-year variability (Campos, Souza Filho e Lima, 2014).

The State of Ceará is characterized by climatic adversities such as high evaporation rates, low precipitation alternated by the high potential for short extreme intense rainfall, and shallow soils, most above crystalline rock basement (Campos, 2014).

In this region, the rainy season is centered around February/April, with substantial interannual variability. It is associated with the seasonal migration of a lower-tropospheric confluence axis over the eastern tropical Atlantic, as known as the Intertropical Convergence Zone (ITCZ) (Uvo *et al.*, 1998; Vasconcelos Junior, Jones e Gandu, 2018).

The ITCZ is closely related to the Sea Surface Temperature (SST). It is usually located on, or close to, the high SST. Therefore, a relationship exists between the general distribution of SST in the Tropical Atlantic and rainfall in NEB. It has been studied that warmer waters in the Tropical South Atlantic and colder waters in the Tropical North Atlantic are associated with rainy years in the NEB (Utida *et al.*, 2019; Uvo *et al.*, 1998).

Given the scarcity and uncertainty about water availability, several water infrastructure improvements have been implemented over the years at NEB, including the Castanhão dam. Appreciably larger storages are required for a given draft, and reliability, found within continental areas related to arid and semi-arid climate type, typically having storage requirements much higher than for other climate types (McMahon *et al.*, 2007).

The Castanhão dam is located in the vicinity of Jaguaribara, at Ceará State. It has a storage capacity of 6.7 billion cubic meters of water, and its watershed area is about 45,000 km², making it one of the largest reservoirs in Latin America. Due to this region's climatic conditions, this reservoir has enormous importance to Ceará, contributing to control of droughts and floods, used as water reserves for human and animal supply, and irrigated agriculture.

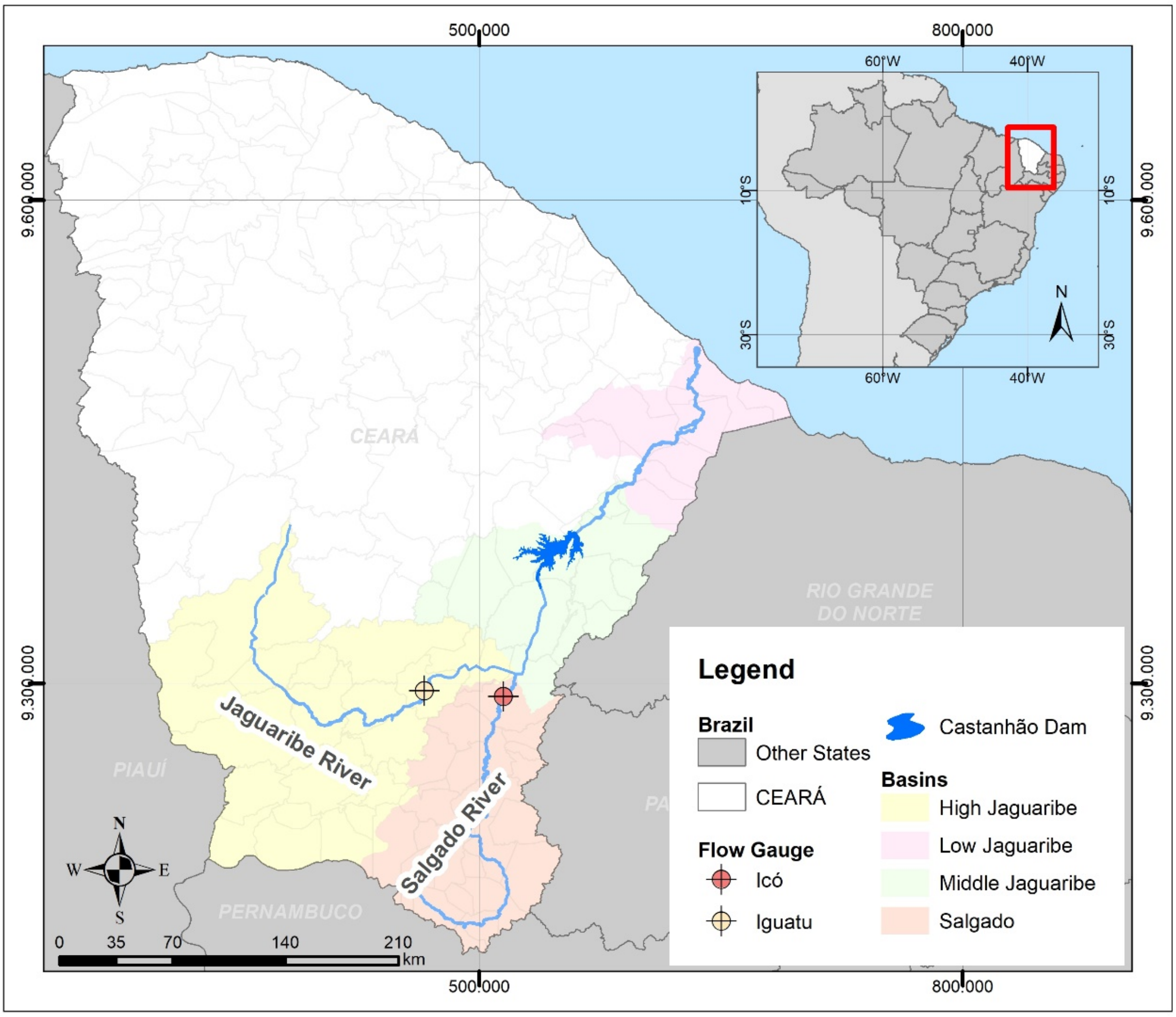
Two main rivers flow into the reservoir: Rio Jaguaribe and Rio Salgado. The regime of its rivers is characterized by intra-annual intermittency (Campos e Studart, 2008).

Jaguaribe River is 633 km long, with a 74,000 km² catchment, draining 81 municipalities, the equivalent of about 50% of the state territory (Dias, Rv e Maia, 2009; Fernandes *et al.*, 2017), and was considered the "biggest dry river of the world" until the 1980s, when it was perennialized by the hydraulic equipment installed in the Orós Reservoir (Monte, 2008).

The mean annual precipitation in the basin is close to 700 mm (Campos, Souza Filho e Lima, 2014), ranging from 400 mm in the inland areas to 800 mm at the coast. In the upper valley, in the Cariri region, the annual precipitation reaches 1000 mm.

Salgado River spring is located at the border of Pernambuco and Ceará states, and it has 308 km long with a 13,450 km² catchment over 23 municipalities. Its basin is formed 85% by crystalline rocks and 15% by sedimentary rocks(Ribeiro, 2017).

Figure 2 - Location of Castanhão Dam and rivers Jaguaribe and Salgado in Ceará.

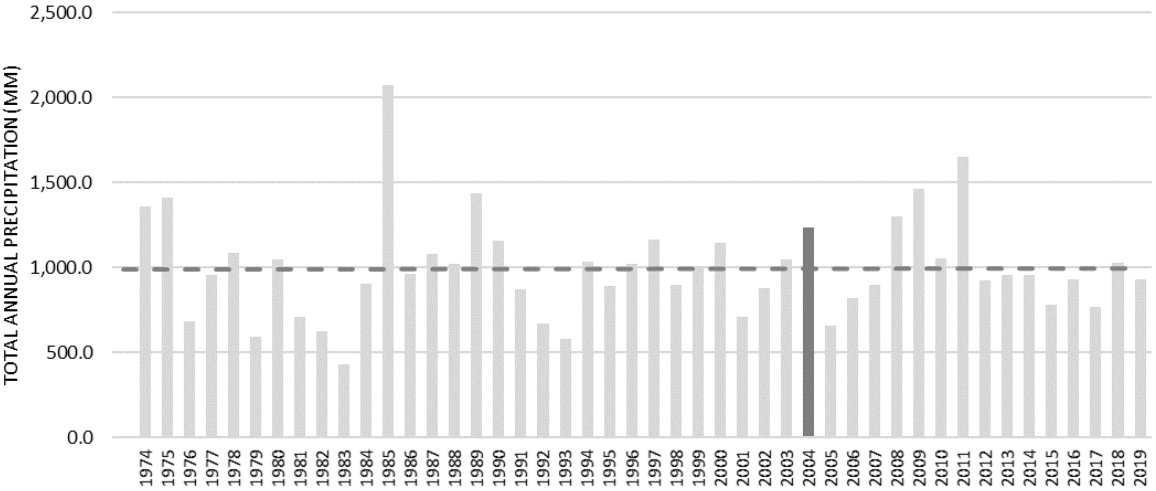


Source: Author (2021).

The flood event in the first semester of 2004 in the State of Ceará was responsible for providing a volume of water capable of filling the Castanhão Reservoir, despite its huge capacity. Such an event arouses curiosity in the scientific community about the singularities of that specific rainy season.

Such an extreme event is to be assumed, at first, that it was caused by a very large amount of rain throughout the year. However, the available data shows that the total rainfall in 2004 year was not one of the highest ones: its values were only slightly above the historical average, as demonstrated by the Iguatu rain gage station historical records (Figure 3). Therefore, only the analysis of the annual amount of rain is not able to represent the real event that occurred.

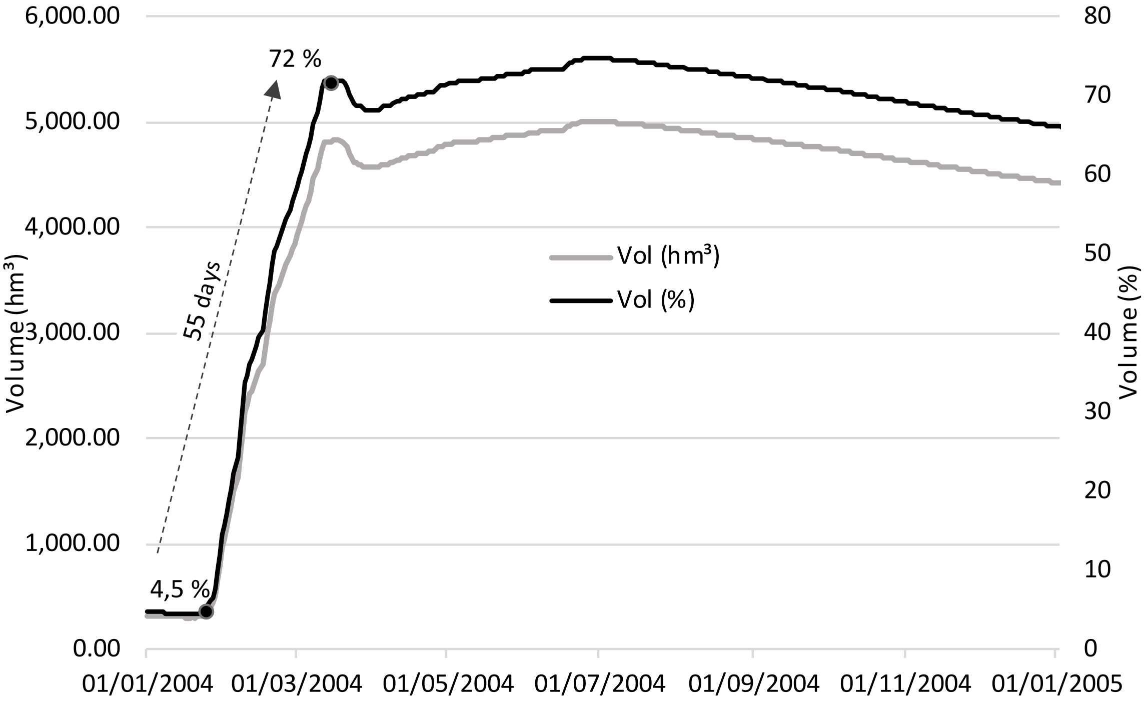
Figure 3 - Historical records of annual precipitation from 1973 to 2019 at Iguatu rain gauge station.



Source: Author (2021).

In 2004, the volume of water stored in Castanhão Reservoir increased from 4,5% to 72% of its storage capacity after exactly 55 days of increasing volumes considered at the dam, from January 22nd to March 16th , as shown in Figure 4, according to the data provided by the Ceará Meteorological Foundation (FUNCEME). That storage could have been even greater, had it not been partly lost by the opening of its floodgates in that time.

Figure 4 - Storage volume in hm³ and the percentage of the total storage capacity in Castanhão Reservoir between January of 2004 and January of 2005.



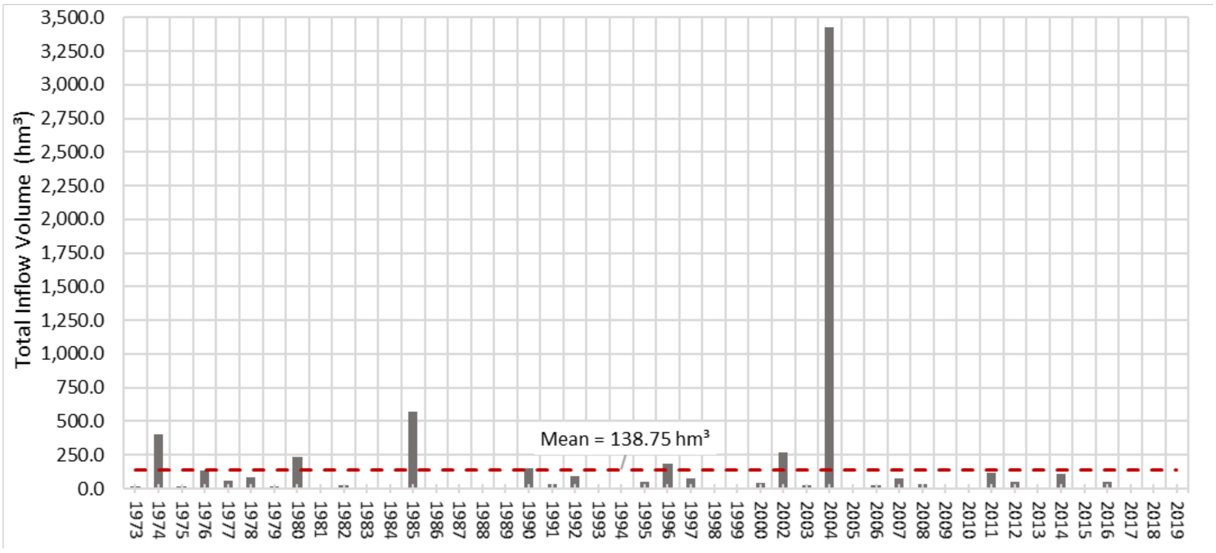
Source: Author (2021).

In other words, in less than 2 months, more than 4,500 million cubic meters of water flowed into Castanhão Reservoir. Such volume variation was expected to happen at least during the main rainy season, wich runs until May.

Several studies were carried out about this rainy season to investigate the climatic phenomena that occurred at that time. The work of Alves et al. (2006) found that January and February showed anomaly moisture flows incoming from Amazônia and the Atlantic Ocean. In some NEB regions, the authors concluded that the rainfall in January 2004 was the highest recorded in the last 40 years and reached more than 100% of the average annual rain in some places in this single month.

This exceptionality that occurred in this period can also be confirmed with the data presented in Figure 5, which shows the historical records, from 1973 to 2019, of the total inflow volume at Jaguaribe River, considering only January and February, where 2004 records clearly stand out from the rest, with a value about 25 times higher than the historical average.

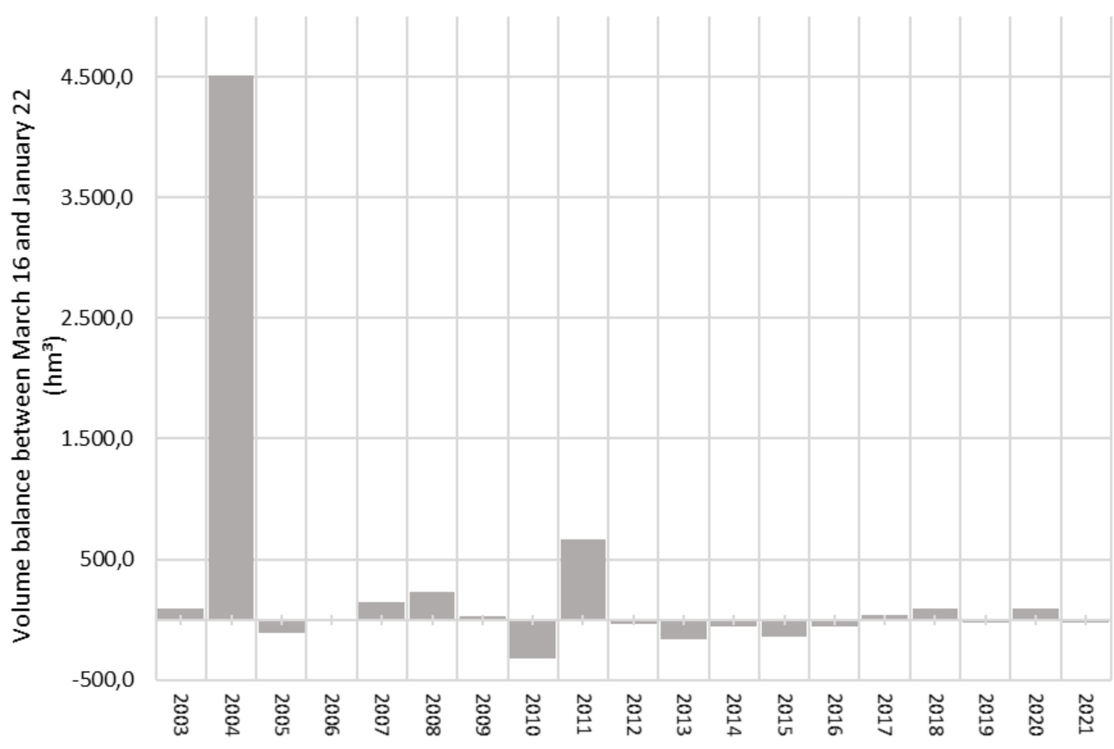
Figure 5 - Total Inflow Volume considering January and February at Jaguaribe River, according to the river gauges station of Iguatu.



Source: Author (2021).

For comparison purposes, Figure 6 shows the total volume balance at the Castanhão Dam during the same period considered (January 22nd to March 16th) from 2003 to 2021. These data are essential to analyze that there were exceptional issues both in the volumes registered and in the period of the year that occurred, which makes the 2004 season a very peculiar event in these terms.

Figure 6 – Volume inflow balance registered at Castanhão Reservoir between March 16th and January 22nd, from 2003 to 2021.



Source: Author (2021).

However, when considered from the global annual perspective, the records at the river gauge station of Iguatu indicates that the discharge in 2004 is only the third-highest annual record since the 1970s, surpassed by the 1974 and 1985 maximum annual peak.

The real impacts of the 2004 rain season, in addition to the rapid filling of Castanhão Reservoir, were the flooding of several locations and dam breaks throughout the entire NEB territory.

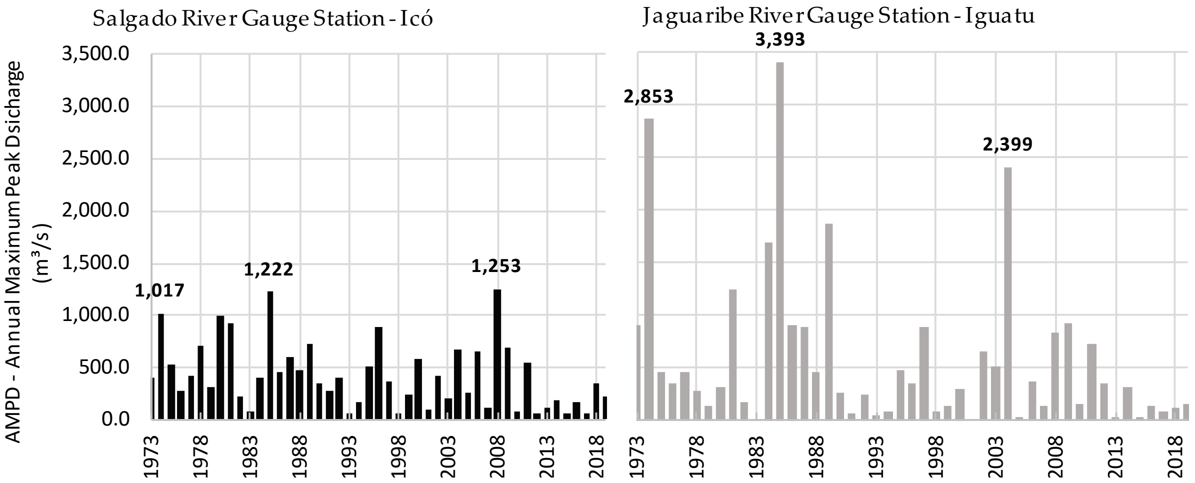
The data analysis result suggests that other parameters should be analyzed for a more comprehensive characterization of a flood event, in addition to the traditional hydrological approach based on the peak flow annual analysis, that under a shallow look, do not justify the dimension of what happened in this event.

Therefore, the research seeks to study the flood events in this basin, from an integrated perspective both of peak flows and volumes recorded during the event, based on a copula frequency analysis.

To set up a statistical model for flow distributions, long and reliable historical continuous observations are needed. The Brazilian National Water Agency (ANA), known for its high data quality standards, made available the data for two river gauge stations located in the main rivers that flow to Castanhão dam. The Icó river gauge station is located at Rio Salgado with a 12,000 km² catchment, and the Iguatu river gauge station is located at Rio Jaguaribe, draining an area of 21,000 km².

These historical data, starting in the year 1973, were used in the study. Some of the flow data series had missing records in 2001 that were not considered in the statistical analysis. With the daily flow records, it was possible to obtain the Annual Maximum Peak Discharges (AMPD) for each station, as shown in Figure 7.

Figure 7 - AMPD from 1973 to 2019 for both river gauge stations.

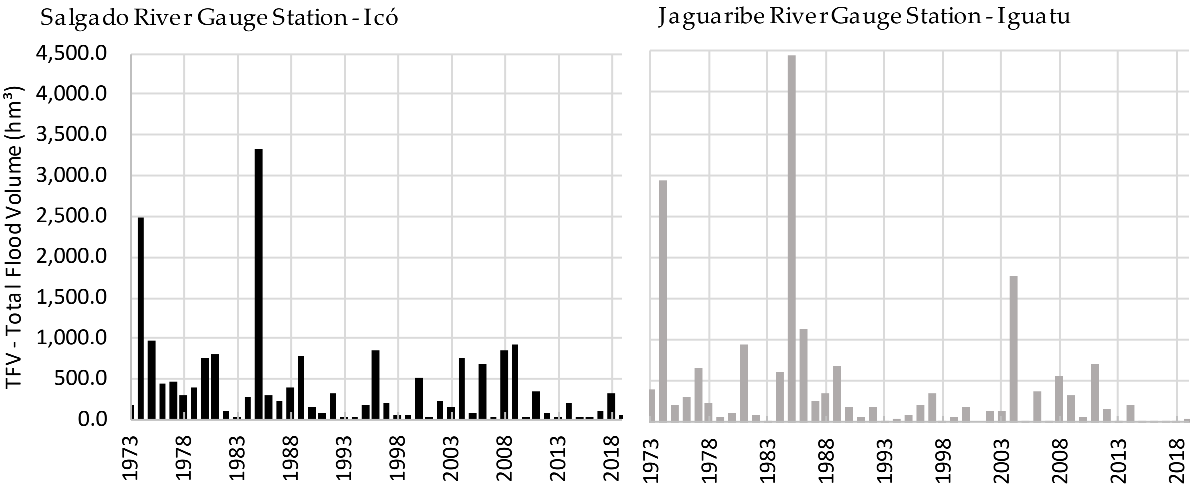


Source: Author (2021).

The graph shows that both river gauge stations registered very high peak discharge in 1974 and 1985. Additionally, high peak discharges also occurred in 2008, at the Icó station, and in 2004, at the Iguatu station. The difference between peak discharges at the two river gauge stations is justified by the difference between drainage areas, which is almost twice at the Iguatu station.

The second variable analyzed was the Total Flood Volume (TFV), defined as the volume from the beginning of each flood event until the time of occurrence of the respective AMPD. This consideration was made to ensure that the volume considered is bounded to the maximum flood event analyzed, defined by the AMPD and to due to the fact that the rivers in the NEB region have pattern of intermittent regime, causing their flow to be null during non-rainy periods, growing after the start of the rainy season and quickly reverting to zero when the rains ceases. The volumes achieved show that 1974 and 1985 also stand out as the maximum values (Figure 8).

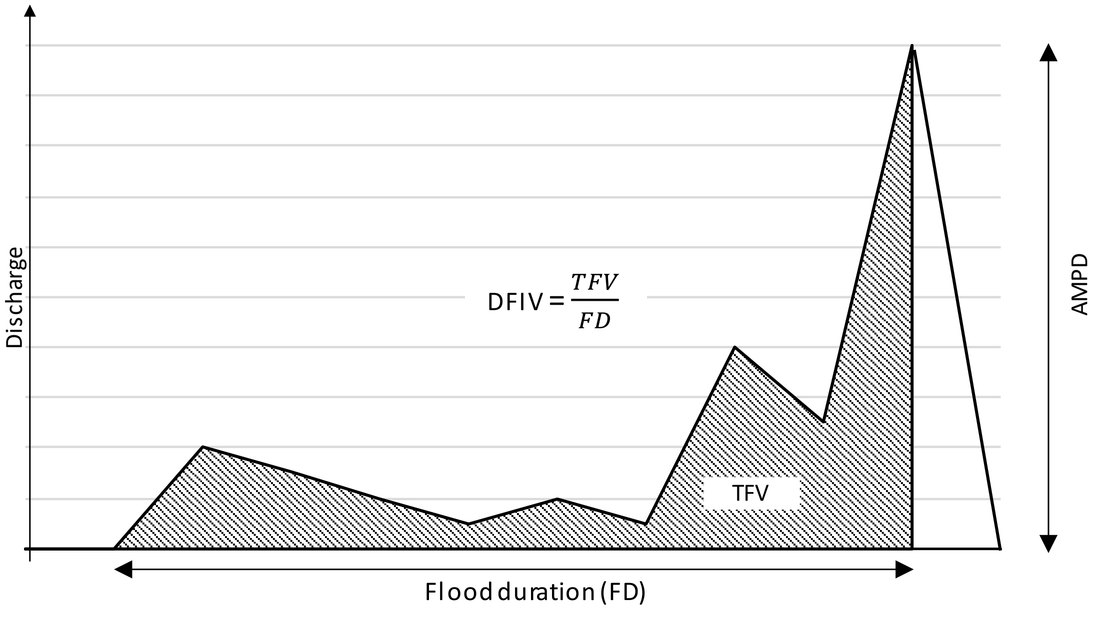
Figure 8 - TFV from 1973 to 2019 for both river gauge stations.



Source: Author (2021).

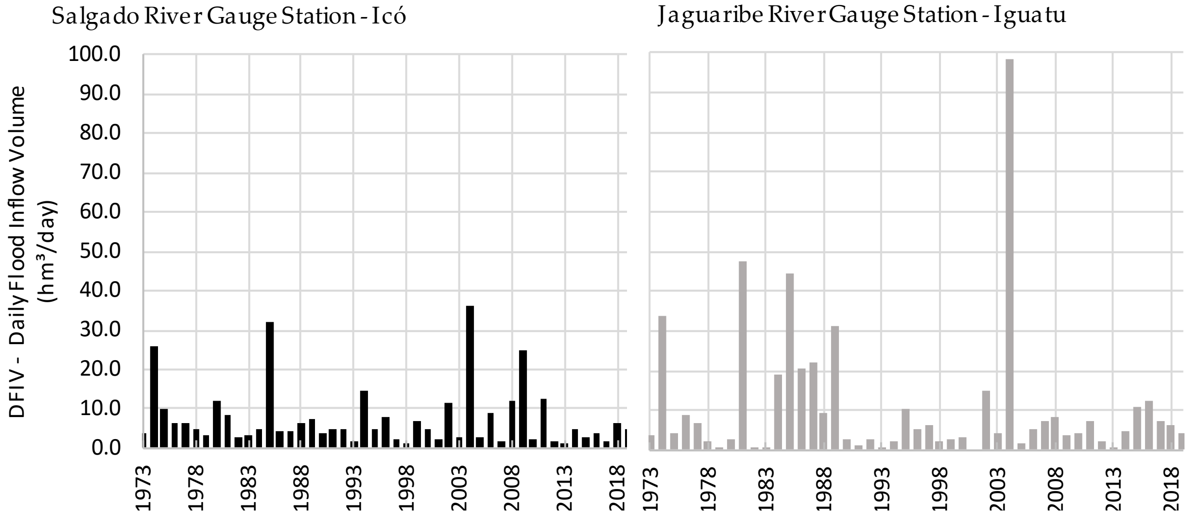
To better characterize each hydrological event in all its aspects, and to investigate the particularity of 2004, a third variable was considered: the ratio between TFV and duration in days of that event. Such a variable can be understood as the average of the Daily Flood Inflow Volume (DFIV). Figure 9 schematizes the variables analyzed. The values of DIFV are shown in Figure 10.

Figure 9 - Definition of the variables used



Source: Author (2021).

Figure 10 - DFIV from 1973 to 2019 for both flow gauges.



Source: Author (2021).

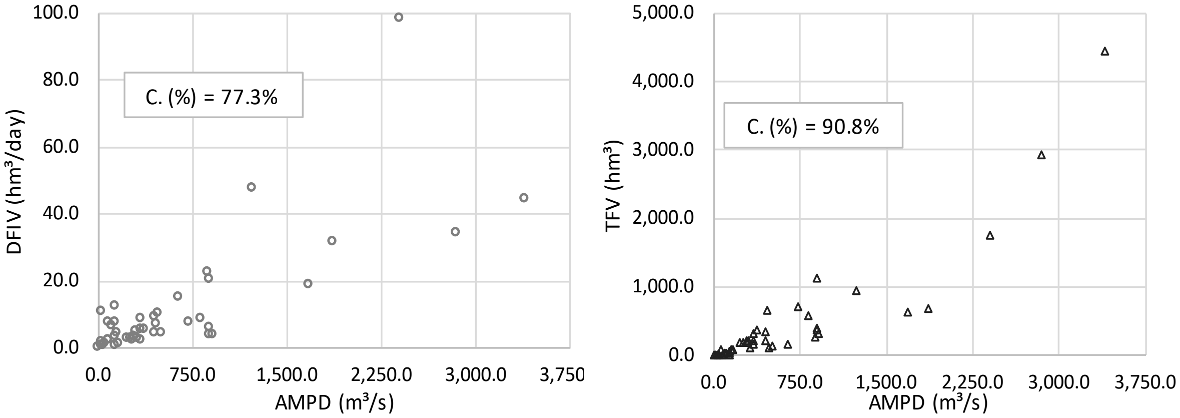
From this perspective, the graphs show interesting results over the year 2004, with the highest values in both gauges. The DFIV value in 2004 at Iguatu river gauge station is clearly out of range. In hydrological terms, it can be understood from this variable that during this event in 2004, there was a remarkable sequence of flows in a short time, causing runoff with maximum intensity, given the conditions of complete soil saturation and, therefore, minimal losses.

The situation can be compared to flash flood events that may generate soil crusting. The areas next to the stream channels become saturated due to the shallow soils. Continuing rainfall causes the water table to rise to the ground surface, and the lower catchment slopes become saturated with further rain flowing overland to the river channel. Runoff rates often far exceed those of other flood types due to the rapid response of the catchments to intense rainfall, modulated by soil moisture and soil hydraulic properties (Borga *et al.*, 2011; Collier, 2007; Hardy *et al.*, 2016; Ruiz-Villanueva *et al.*, 2012).

These flow conditions combined with the data presented can help to justify the large amount of water drained into the reservoir in such a short time.

The same physical phenomenon defines flood peak, duration, and volume; thus, they should be mutually correlated (Grimaldi e Serinaldi, 2006). The scatter plots between AMPD and DFIV and between AMPD and TFV displayed in Figure 11 suggest the dependency between each two coupled variables.

Figure 11 - Data Correlation Scatter Plot DFIV x AMPD and TFV x AMPD. In the figure, C stands for the correlation coefficient.



Source: Author (2021).

5 RESULTS

5.1 Flood Statistical Characteristics

To better comprehend the whole flood event, it is essential to understand the relationship among the variables previously defined. The analysis of the descriptive statistics of flood events (Table 1) showed in general that the coefficient of variation (CV) for all variables is higher at the Iguatu gauge. In other words, it has a high steepness of the flood frequency curve since it is calculated as the standard deviation of the annual series divided by the mean.

This CV can be considered a value well above the average of the world's basins compared to the global perspective. High CVs may be due to nonlinear runoff generation processes, particularly threshold behavior, prevalent in semi-arid regions (Blöschl e Sivapalan, 1997; Chen *et al.*, 2014; Grafton e Hussey, 2011).

Table 1 - Descriptive Statistics of flood events for both flow gauges

River gauge station	**Nº Years**	**Annual Maximum Peak Discharge, AMPD (m³/s)**			**Total Flood Volume, TFV (hm³)**			**Daily Flood Inflow Volume, DFIV (hm³/day)**		
		Máx	**Mean**	**CV**	**Máx**	**Mean**	**CV**	**Máx**	**Mean**	**CV**
Iguatu	46	3,393.3	589.8	1.3	4,451.9	425.6	1.9	98.2	10.9	1.6
Icó	46	1,253.5	431.9	0.7	3,315.7	418.2	1.5	36.4	7.6	1.0

Source: Author (2021).

5.2 Estimation Distributions

Even with studies showing a good correlation of all variables, it was decided to focus on the distribution of the joint of only DFIV and AMPD, given that the main objective is to investigate the event of the year 2004, where the first variable has excellent prominence.

Several authors have considered the joint modeling of flows and volumes (Adamson, Metcalfe e Parmentier, 1999; Singh e Singh, 1991; Yue, Ouarda e Bobée, 2001). In these studies, the marginal distributions were considered the same for both random variables involved in the analysis; Yue (2001) used Gamma Distribution. However, the marginal distributions of flows and volumes may differ, which happens to be the case in this study.

The bivariate copulas were used to determine the best-fitted copula for modeling bivariate joint distribution between flow peaks and volumes, aiming at obtaining the corresponding joint return periods, which are essential for evaluating and predicting the regional flood magnitude. According to Sections 3.1 and 3.2, a marginal distribution was chosen for each parameter and each gauge station as shown in Table 2 and one copula function was selected for each river gauge station. The estimated parameters and values of the goodness-of-fit test (AIC) for the best-fitted copula are listed in Table 3. For each river gauge station, the Survival Gumbel was chosen as the best fit.

Table 2 – Selected Marginal distributions and parameters

Time Series	**Annual Maximum Peak Discharge, AMPD**				**Daily Flood Inflow Volume, DFIV**			
	Marginal	**Par1**	**Par2**	**AIC**	**Marginal**	**Par1**	**Par2**	**AIC**
Iguatu	Weibull	0,85	540,07	680,76	Weibull	0,81	10.058.881,15	1582,25
Icó	Weibull	1,40	474,28	647,20	Lognorm	15,48	0,85	1540,05

Source: Author (2021).

Table 3 - Selected copula families, Kendall tau (t) and copula parameters

Time Series	**Copula**			
	Selected Family	**Par**	**Kendall's t**	**AIC**
Iguatu	Survival Gumbel	2,56	0,61	-46,85
Icó	Survival Gumbel	2,26	0,56	-41,04

Source: Author (2021).

5.3 Return Period of Floods

Table 4 and Figure 12 show the calculated return periods. In general, very unusual events were found in 1974, 1985 and 2004.

Regarding the univariate analysis, the DFIV that occurred in 2004 in Iguatu presented a return period of 584 years. However, the exceptionality is even more remarkable when the multivariate copula distribution is used, combining DFIV with AMPD, where the value found reaches almost 995 years.

This joint event of extremely low frequency can only be found due to the copula function. The event with the two parameters perspective had its return period almost doubled regarding the single variable analysis.

Table 4 - Return Period in years of AMPD and DFIV for univariate distribution. Joint AMPD or DFIV and AMPD & DFIV for the bivariate distribution.

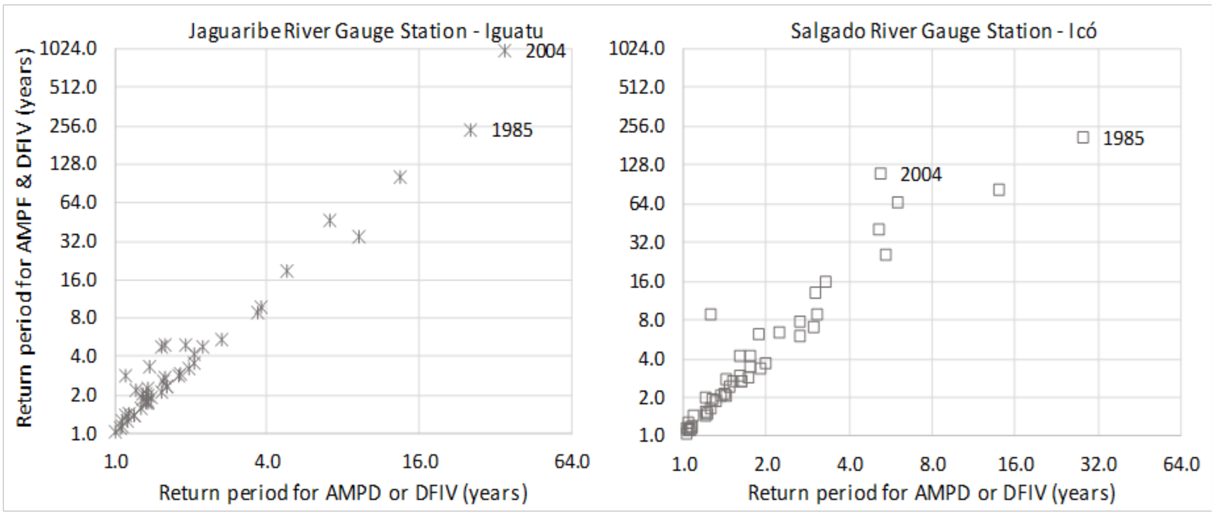
Event	**Jaguaribe River Gauge Station - Iguatu**				**Salgado River Gauge Station - Icó**			
Year	**AMPD**	**DFIV**	**Or**	**&**	**AMPD**	**DFIV**	**Or**	**&**
1973	4.7	1.5	1.5	4.8	2.3	1.5	1.5	2.5
1974	63.0	14.8	13.5	102.7	18.3	34.1	13.9	81.3
1975	2.4	1.6	1.6	2.6	3.2	4.3	2.6	6.0
1976	2.0	2.4	1.8	2.9	1.6	2.5	1.5	2.7
1977	2.4	2.1	1.8	2.9	2.4	2.5	1.9	3.3
1978	1.8	1.3	1.3	1.8	5.8	1.9	1.9	6.3
1979	1.4	1.1	1.1	1.4	1.8	1.4	1.3	1.9
1980	1.9	1.4	1.4	2.0	16.9	6.1	5.4	25.6
1981	7.5	34.3	7.1	46.7	12.9	3.5	3.3	15.9
1982	1.4	1.1	1.1	1.5	1.4	1.3	1.2	1.5
1983	1.0	1.0	1.0	1.0	1.1	1.5	1.1	1.5

Table 4 - Return Period in years of AMPD and DFIV for univariate distribution. Joint AMPD or DFIV and AMPD & DFIV for the bivariate distribution.

1984	13.9	5.3	4.8	18.6	2.2	1.8	1.6	2.6
1985	122.0	28.5	25.6	238.3	42.8	59.5	28.3	**207.4**
1986	4.7	5.9	3.7	8.8	2.6	1.7	1.6	2.9
1987	4.6	6.7	3.8	9.6	3.9	1.6	1.6	4.2
1988	2.4	2.5	2.0	3.2	2.8	2.4	2.0	3.6
1989	17.8	12.4	9.3	34.9	6.2	2.9	2.6	7.7
1990	1.7	1.4	1.3	1.8	1.9	1.5	1.4	2.1
1991	1.2	1.2	1.1	1.2	1.6	1.8	1.4	2.1
1992	1.6	1.4	1.4	1.8	2.2	1.8	1.6	2.6
1993	1.1	1.1	1.1	1.1	1.0	1.1	1.0	1.2
1994	1.2	1.3	1.2	1.4	1.3	8.7	1.3	8.8
1995	2.5	2.8	2.1	3.5	3.0	1.9	1.7	3.4
1996	2.0	1.8	1.6	2.3	10.7	3.2	3.0	13.1
1997	4.6	1.9	1.9	4.9	2.0	1.2	1.2	2.0
1998	1.2	1.3	1.2	1.4	1.1	1.0	1.0	1.1
1999	1.4	1.4	1.3	1.6	1.5	2.6	1.4	2.8
2000	1.8	1.5	1.4	1.9	3.8	1.8	1.7	4.2
2002	3.2	4.0	2.7	5.4	2.4	5.4	2.2	6.3
2003	2.6	1.7	1.6	2.8	1.4	1.3	1.2	1.5
2004	35.6	584.8	34.7	**994.8**	5.3	89.1	5.2	109.2
2005	1.1	1.2	1.1	1.2	1.6	1.3	1.3	1.7
2006	2.1	1.8	1.6	2.4	4.7	3.8	3.0	7.2
2007	1.4	2.2	1.3	2.3	1.1	1.1	1.1	1.2
2008	4.2	2.4	2.2	4.8	48.9	6.2	6.0	65.8
2009	4.8	1.6	1.6	4.9	5.3	30.2	5.1	40.3
2010	1.4	1.7	1.3	1.8	1.1	1.2	1.1	1.2
2011	3.6	2.2	2.0	4.2	3.4	6.6	3.0	8.9
2012	2.0	1.4	1.3	2.1	1.1	1.1	1.1	1.1
2013	1.1	1.1	1.1	1.1	1.2	1.0	1.0	1.2
2014	1.8	1.7	1.5	2.1	1.3	1.9	1.3	2.0
2015	1.1	2.9	1.1	2.9	1.1	1.3	1.0	1.3
2016	1.4	3.3	1.4	3.3	1.3	1.5	1.2	1.6
2017	1.2	2.1	1.2	2.2	1.1	1.1	1.0	1.1
2018	1.3	2.0	1.3	2.1	2.0	2.4	1.7	2.9
2019	1.4	1.7	1.3	1.8	1.4	1.9	1.4	2.1

Source: Author (2021).

Figure 12 - Return Periods for AMPD or DFIV and AMPD & DFIV time series.



Source: Author (2021).

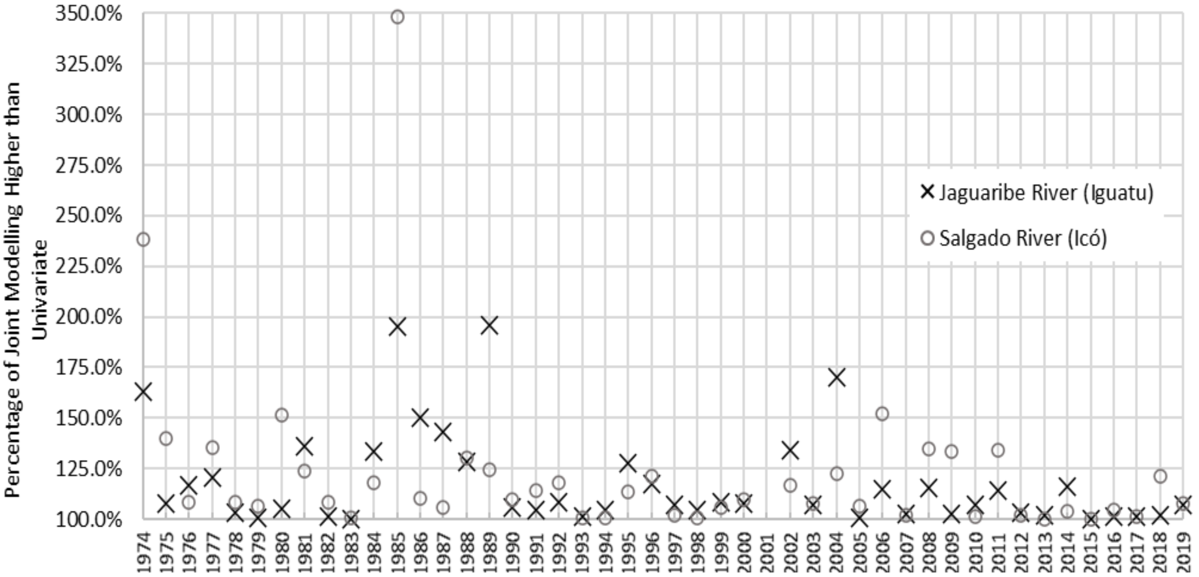
The results of the bivariate analysis can be compared to the results of the univariate analysis. It can be seen an information gain by using this methodology in comparison with the traditional univariate results. Figure 13 shows the difference in percentual terms between the return periods of the joint distribution analysis concerning the maximum return period obtained in the analysis of the univariate parameters, either with the peak or the volume, for the two flow gauges studied.

In general, the higher the point in the graph, the more significant the difference between the bivariate and univariate methodology, so more considerable is the inaccuracy of using just one parameter to quantify the magnitude of the flood event.

Most values are between 100% and 125%, which means that the bivariate distribution did not make much difference in this case than the isolated analysis of events. Still, especially for the most notable events (1974, 1985, 2004), the methodology proved very relevant, as the values were higher. It is shown that the most significant difference is found in Icó, in 1985, where the value of the return period of the joint distribution (207.4 years) is practically 3.5 times greater than the return period obtained in the univariate analysis (59.5 years for the DFIV).

Although not among the maximum values of this comparison, the year 2004 presents a considerable high difference in the return period using the copula function.

Figure 13 - Difference in percentual between the joint bivariate return periods against the univariate results.



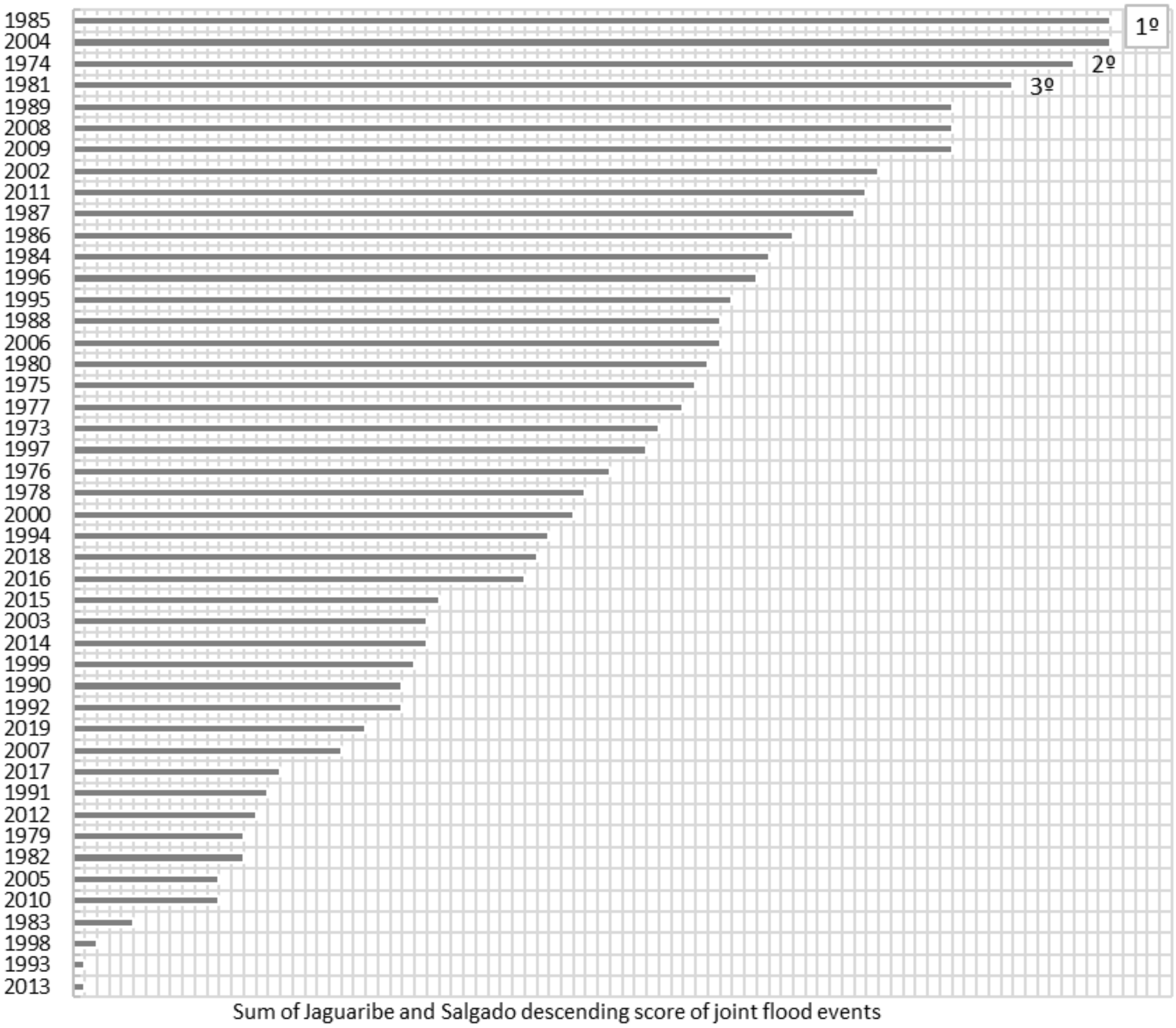
Source: Author (2021).

Figure 14 shows the comparison between the two basins and the ranking of the year's events in terms of flood severity.

To build this graphic, a score was assigned to each variable according to their position in relation to the events exceptionality. The ranking (1 to 46) of the combined AMPF & DFIV events in each year was sorted in descending order, and its value was added for each flow gauge. Thus it is possible to group all the variables and gauges so that the years are classified, from the rainiest to the driest.

The events in 1985 and 2004 are tied to the lowest classified years, which were the most extraordinary droughts identified in the region (1993, 1998, and 2013).

Figure 14 - Descending joint distribution return period ranking for annual events of AMPF & DFIV in both flow gauge



Source: Author (2021).

6 DISCUSSION AND CONCLUSION

In this study, a copula-based methodology is presented and applied to the frequency analysis of flood events. In a real case study: the two main rivers that flow to Castanhão Reservoir.

In the bivariate approach, flood flow characteristics such as annual flood peak flow, flood volume, and flood duration were considered. The correlation between those variables was statistically significant and, accordingly, able to be considered in a bivariate approach.

The year 2004 was an unusual hydrological year, with both positive consequences (such as filling the Castanhão reservoir) and negative ones (such as the rupture of dams and floods in the NEB).

In 2004, the characteristics of some of the variables, like the precipitation, the peak flood, and the total annual volume, were not as high as in other rainy years, and they could remain almost unnoticed. However, it was found that the floods were extremely bulky over a short period, based on the first two months.

With the marginal probability distributions established and based on the bivariate copula functions, each event's return period was calculated. The univariate analysis showed that the 2004 discharge event has a very low frequency, with a return period of about 585 years. However, the joint analysis of peak discharge and flood volume using the copulas resulted in a return period of 985 years, almost 170% higher than the one given by the univariate analysis.

These differences in the presented return periods indicate that, from the perspective of a safety analysis, a particular hydraulic construction placed in this basin could be subject to an almost twice as high risk when taking into account multivariate analysis.

Thus, copula functions can model the complex nature of hydrological events better than the simplified methodology most commonly used that considers only univariate analysis.

Still, it was proved that the event that occurred in the Jaguaribe River basin had greater relevance than that of the Salgado river basin, in terms not only of the discharges and volumes (which were already expected to be greater in Jaguaribe due to the larger area of the basin) but also of the return periods, that were much higher in Jaguaribe River.

With this result, it is possible that future work can be directed to that specific location of the Rio Jaguaribe basin to investigate the phenomena that occurred in the period that was decisive for the filling of Castanhão Reservoir.

Therefore, this work's main contribution was to show that the traditional methods of modeling floods, which usually consider only peak flows, may underestimate or even conceal some extreme events, such as in the case study. By coupling other variables to those flows via a copulas approach, it was possible to understand the exceptionality of the events and, consequently, understand the importance of coupling variables for flood prediction and the design of hydraulic infrastructures.

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