Block Diagonalization For Multicell Multiuser MIMO Systems with Other-Cell Interference

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***Abstract*— We consider the Block Diagonalization (BD) approach assuming multicell multiuser Multiple-Input Multiple-Output (MIMO) systems. For such systems, conventional BD (cBD) eliminates the intra-cell interference but neglects effects related to other cells, i.e., the Other-Cell Interference (OCI). Consequently, an enhanced BD (eBD) was proposed in the literature, which accounts for the presence of the OCI. However, both cBD and eBD suffer from dimensionality restrictions at the Base Station (BS). In this paper, we propose the iterative BD (iBD), which reduces the dimensionality restrictions and accounts for the presence of the OCI as well. It is found that iBD and eBD are equivalent whenever the number of transmitted data streams to a user is equal to the number of its receive antennas. Otherwise, iBD outperforms both eBD and cBD. Detailed simulation results are presented to evaluate the performance of all considered BD approaches.**

***Keywords*— Block diagonalization, MIMO, other-cell interference, intra-cell interference.**

I. Introduction

The network densification is considered to be among the key solutions to increase the capacity and throughput of current and future cellular networks. The conventional macro-based cellular networks are expected to be densified by low-power nodes in order to handle the rapidly increasing number of users and high Quality-of-Service (QoS) demand [1]. However, the dense network performance is limited by intra-cell and inter-cell interference.

In order to completely eliminate the intra-cell interference, a possible approach is to simply use the conventional BD (cBD) algorithm [2]. However, cBD targets Broadcast networks, i.e., with a single cell and multiple users. Therefore, with multicell networks, the users will suffer from the inter-cell interference, which can also be denoted as Other-Cell Interference (OCI). One possible solution, which is presented in [3], is to enhance the cBD algorithm so that it would account for the OCI presence.

This paper is organized as follows. We first present the system model in Section II. In Section III, we present the achievable throughput of block diagonalization, with each different approach presented in a different subsection, namely: conventional BD (cBD), enhanced BD (eBD) and iterative BD (iBD). In Section IV we present the numerical results. Finally, we conclude this work in Section V.

II. System Model

We consider a multicell multiuser Multiple-Input Multiple-Output (MIMO) system[1] in which there are M cells sharing the same resources and thus interfering with each others' transmission. The Base Station (BS) of cell $m, m = 1, \ldots, M$, is equipped with N_t transmit antennas and serves K local users or mobile stations (MSs). See Figure 1.

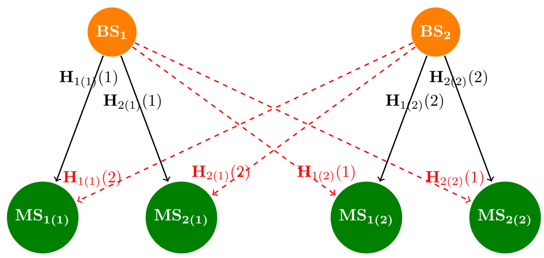


Fig. 1. Multicell Multiuser MIMO System $[M = 2, K = 2]$.

Let us define by $i(m)$ the i^{th} user in cell m and by N_r the number of receive antennas. Let us also define by $\mathcal{K}$ the set of all users, i.e., $\mathcal{K} \overset{\text{def}}{=} \{i(m) | i \in \{1, \ldots, K\}, m \in \{1, \ldots, M\}\}$. Throughout this paper, $i, j, l \in \{1, \ldots, K\}$ while $m, n \in \{1, \ldots, M\}$. We assume that user $i(m)$ receives $d_{i(m)}$ data streams, where $1 \le d_{i(m)} \le \min\{N_r, N_t\}$. Assuming a linear channel model, the received signal at user $i(m)$ can be written as

$$\mathbf{y}_{i(m)} = \underbrace{\mathbf{H}_{i(m)}(m)\mathbf{T}_{i(m)}\mathbf{s}_{i(m)}}_{\text{desired signal}} + \underbrace{\sum_{j, j \neq i} \mathbf{H}_{i(m)}(m)\mathbf{T}_{j(m)}\mathbf{s}_{j(m)}}_{\text{intra-cell interference}}$$
$$+ \underbrace{\sum_{n, n \neq m} \sum_{l} \mathbf{H}_{i(m)}(n)\mathbf{T}_{l(n)}\mathbf{s}_{l(n)} + \mathbf{z}_{i(m)}}_{\text{inter-cell interference/OCI plus noise}}, \qquad (1)$$

where $\mathbf{y}_{i(m)} \in \mathbb{C}^{N_r}$, $\mathbf{H}_{i(m)}(n) \in \mathbb{C}^{N_r \times N_t}$ represents the channel matrix from the n^{th} BS to user $i(m)$, $\mathbf{T}_{i(m)} \in \mathbb{C}^{N_t \times d_{i(m)}}$ is the transmit beamforming matrix of user $i(m)$, $\mathbf{s}_{i(m)} \in \mathbb{C}^{d_{i(m)}}$ is the transmit data vector to user $i(m)$, and $\mathbf{z}_{i(m)} \in \mathbb{C}^{N_r}$ denotes the white Gaussian noise with distribution $\mathcal{N}(0, \sigma^2_{i(m)}\mathbf{I})$.

[1]Notations: Upper/lower boldface letters are used for matrices/vectors. The notations $(\cdot)^T$ and $(\cdot)^H$ indicate the transpose and the complex conjugate transpose, respectively. $|\cdot|$ indicates the determinant operation. $\dim(\mathbf{A})$ indicates the dimension of $\mathbf{A}$. $\mathbf{A} \propto \mathbf{B}$ is used to denote that $\mathbf{A}$ is proportional to $\mathbf{B}$. $\text{blockdiag}\{\cdots\}$ denotes the block-diagonal of the provided matrices/vectors.

We consider linear receive beamforming. The received symbols of user $i(m)$ can be written as

$$\hat{\mathbf{s}}_{i(m)} = \mathbf{R}_{i(m)}^H \mathbf{y}_{i(m)}, \tag{2}$$

where $\mathbf{R}_{i(m)} \in \mathbb{C}^{N_r \times d_{i(m)}}$ represents the receive beamforming matrix of user $i(m)$. The OCI plus noise covariance matrix of user $i(m)$ is given by

$$\boldsymbol{\Upsilon}_{i(m)} = \sum_{n,n\neq m} \sum_{l} \mathbf{H}_{i(m)}(n)\mathbf{T}_{l(n)}\mathbf{T}_{l(n)}^H\mathbf{H}_{i(m)}^H(n) + \sigma_{i(m)}^2\mathbf{I}. \tag{3}$$

III. Block Diagonalization (BD)

Theoretically, BD can be interpreted as an equivalent transmit zero-forcing (ZF) algorithm for MIMO systems, but in which the interference among streams of a same user has to be canceled at the receiver side. The main objective is to completely eliminate the intra-cell interference by forcing each user to transmit on the null space of all other users on the same cell. Assuming $\mathbf{R}_{i(m)}\mathbf{H}_{i(m)}(m)\mathbf{T}_{j(m)} = 0, \forall i(m) \neq j(m), i(m), j(m) \in \mathcal{K}_m \subset \mathcal{K}$, the achievable throughput of the m^{th} cell is given by

$$\mathcal{C}_m^{BD} = \max_{\mathbf{T}_m} \log \left| \mathbf{I} + \frac{\mathbf{R}_m^H\mathbf{H}_m\mathbf{T}_m\mathbf{T}_m^H\mathbf{H}_m^H\mathbf{R}_m}{\mathbf{R}_m^H\boldsymbol{\Upsilon}_m\mathbf{R}_m} \right|, \tag{4}$$

where

$$\mathbf{H}_m = \left[\mathbf{H}_{1(m)}^T(m) \quad \dots \quad \mathbf{H}_{K(m)}^T(m)\right]^T.$$
$$\mathbf{T}_m = \left[\mathbf{T}_{1(m)} \quad \dots \quad \mathbf{T}_{K(m)}\right].$$
$$\boldsymbol{\Upsilon}_m = \text{blockdiag}\{\boldsymbol{\Upsilon}_{1(m)}, \dots, \boldsymbol{\Upsilon}_{K(m)}\}.$$
$$\mathbf{R}_m = \text{blockdiag}\{\mathbf{R}_{1(m)}, \dots, \mathbf{R}_{K(m)}\}.$$

In the following, we present three BD approaches to calculate $\mathbf{T}_{j(m)}$ and $\mathbf{R}_{i(m)}, \forall i(m) \in \mathcal{K}$.

A. Conventional BD

The cBD algorithm was first shown in [2]. The transmit beamforming matrix of user $i(m)$ is given by

$$\mathbf{T}_{i(m)} = \mathbf{G}_{i(m)}\mathbf{F}_{i(m)}\mathbf{P}_{i(m)}^{\frac{1}{2}}, \tag{5}$$

where $\mathbf{P}_{i(m)}$ is a diagonal matrix and holds the power allocation, $\mathbf{G}_{i(m)}$ holds the orthogonal basis vectors of the null space obtained from the *intra-cell users' channels*, which is given by

$$\begin{aligned}\mathbf{H}_{i(m)}^{-i} =& [\mathbf{H}_{1(m)}^T(m) \quad \dots \quad \mathbf{H}_{i-1(m)}^T(m) \\ & \mathbf{H}_{i+1(m)}^T(m) \quad \dots \quad \mathbf{H}_{K(m)}^T(m)]^T,\end{aligned} \tag{6}$$

and $\mathbf{F}_{i(m)}$ holds the right singular vectors obtained from the *effective channel* of user $i(m)$ given by

$$\mathbf{H}_{i(m)}^e = \mathbf{H}_{i(m)}(m)\mathbf{G}_{i(m)}. \tag{7}$$

To calculate $\mathbf{G}_{i(m)}$, define the singular value decomposition (SVD) of $\mathbf{H}_{i(m)}^{-i}$ as

$$\mathbf{H}_{i(m)}^{-i} = \mathbf{U}_{i(m)}^{-i}\boldsymbol{\Sigma}_{i(m)}^{-i}[\mathbf{V}_{i(m)}^{-i} \quad \mathbf{G}_{i(m)}], \tag{8}$$

where $\mathbf{G}_{i(m)}$ corresponds to the last $(N_t - l_{i(m)}^{-i})$ right singular vectors, with $l_{i(m)}^{-i}$ defining the rank of $\mathbf{H}_{i(m)}^{-i}$. Therefore, the condition of $(N_t - l_{i(m)}^{-i}) \geq d_{i(m)}$ should be satisfied so that user $i(m)$ would have a sufficient null space. If $N_t \geq (K-1)N_r + d_{i(m)}$, the latter condition is satisfied with probability one.

To calculate $\mathbf{F}_{i(m)}$, define the SVD of $\mathbf{H}_{i(m)}^e$ as

$$\mathbf{H}_{i(m)}^e = \mathbf{U}_{i(m)}^e \begin{bmatrix} \boldsymbol{\Sigma}_{i(m)}^e & 0 \\ 0 & 0 \end{bmatrix} [\mathbf{V}_{i(m)}^{e(1)} \quad \mathbf{V}_{i(m)}^{e(0)}], \tag{9}$$

where $\boldsymbol{\Sigma}_{i(m)}^e$ is a $[l_{i(m)}^e \times l_{i(m)}^e]$ diagonal matrix and $\mathbf{V}_{i(m)}^{e(1)}$ corresponds to the first $l_{i(m)}^e$ singular vectors, with $l_{i(m)}^e$ being the rank of $\mathbf{H}_{i(m)}^e$. Therefore, assuming the values of $\boldsymbol{\Sigma}_{i(m)}^e$ are in a decreasing order, then we choose $\mathbf{F}_{i(m)}$ to be the first $d_{i(m)}$ vectors of $\mathbf{V}_{i(m)}^{e(1)}$, i.e.,

$$\mathbf{F}_{i(m)} = \left(\mathbf{V}_{i(m)}^{e(1)}\right)_{(1:d_{i(m)})}. \tag{10}$$

Furthermore, we choose $\mathbf{R}_{i(m)}$ to be the first $d_{i(m)}$ vectors of $\mathbf{U}_{i(m)}^e$, i.e.,

$$\mathbf{R}_{i(m)} = \left(\mathbf{U}_{i(m)}^e\right)_{(1:d_{i(m)})}. \tag{11}$$

With transmit and receive beamforming matrices calculated as above, the expression given by (4) is reduced to

$$\mathcal{C}_m^{cBD} = \max_{\mathbf{P}} \log \left| \mathbf{I} + \frac{\boldsymbol{\Sigma}^2\mathbf{P}}{\mathbf{R}_m^H\boldsymbol{\Upsilon}_m\mathbf{R}_m} \right|, \tag{12}$$

where $\boldsymbol{\Sigma} = \text{blockdiag}(\boldsymbol{\Sigma}_{1(m)}^e, \dots, \boldsymbol{\Sigma}_{K(m)}^e)$ and $\mathbf{P}$ is a diagonal matrix that holds the optimal power loading, found using the water-filling method [4] on the $\boldsymbol{\Sigma}$ diagonal elements [2].

B. Enhanced BD

As it can be understood from the previous section, the main issue with cBD, besides the dimensionality restrictions, is that it does nothing to reduce the inter-cell interference (or the OCI) effects. This issue has been considered in [3], where the authors enhanced the cBD algorithm such that it accounts for the presence of OCI.

To suppress the OCI effect, user $i(m)$ uses the whitening matrix $\mathbf{W}_{i(m)} = \boldsymbol{\Upsilon}_{i(m)}^{-\frac{1}{2}}$ at the received signal. Define $\mathbf{W}_m = \text{blockdiag}[\mathbf{W}_{1(m)}, \dots, \mathbf{W}_{K(m)}]$. Thus, the expression given by (4) can be written as

$$\begin{aligned}\mathcal{C}_m^{eBD} &= \max_{\widehat{\mathbf{T}}_m} \log \left| \mathbf{I} + \frac{\widehat{\mathbf{R}}_m^H\mathbf{W}_m\mathbf{H}_m\widehat{\mathbf{T}}_m\widehat{\mathbf{T}}_m^H\mathbf{H}_m^H\mathbf{W}_m^H\widehat{\mathbf{R}}_m}{\widehat{\mathbf{R}}_m^H\mathbf{W}_m\boldsymbol{\Upsilon}_m\mathbf{W}_m^H\widehat{\mathbf{R}}_m} \right| \\ &= \max_{\widehat{\mathbf{T}}_m} \log \left| \mathbf{I} + \widehat{\mathbf{R}}_m^H\widehat{\mathbf{H}}_m\widehat{\mathbf{T}}_m\widehat{\mathbf{T}}_m^H\widehat{\mathbf{H}}_m^H\widehat{\mathbf{R}}_m \right|,\end{aligned} \tag{13}$$

[2]Notice that the water-filling method is applied individually on each sub-matrix of $\boldsymbol{\Sigma}$, assuming that the total transmit power of the BS is divided equally between its K users. Clearly, the equal power allocation is a sub-optimal solution. However, this is done to make sure that all K users are allocated for transmission.

where $\widehat{\mathbf{H}}_m = \mathbf{W}_m\mathbf{H}_m$. In the same way as for cBD, the transmit beamforming matrix of user $i(m)$ is given by

$$\widehat{\mathbf{T}}_{i(m)} = \widehat{\mathbf{G}}_{i(m)}\widehat{\mathbf{F}}_{i(m)}\widehat{\mathbf{P}}^{\frac{1}{2}}_{i(m)}. \quad (14)$$

The $\widehat{\mathbf{G}}_{i(m)}$ matrix is calculated similar to (8) from

$$\widehat{\mathbf{H}}^{-i}_{i(m)} = [\widehat{\mathbf{H}}^T_{1(m)} \quad \dots \quad \widehat{\mathbf{H}}^T_{i-1(m)} \quad \widehat{\mathbf{H}}^T_{i+1(m)} \quad \dots \quad \widehat{\mathbf{H}}^T_{K(m)}]^T, \quad (15)$$

where $\widehat{\mathbf{H}}_{i(m)} = \mathbf{W}_{i(m)}\mathbf{H}_{i(m)}(m)$. The $\widehat{\mathbf{F}}_{i(m)}$ and $\widehat{\mathbf{R}}_{i(m)}$ matrices are calculated using (10) and (11), respectively, from the effective channel of user $i(m)$ that is given by

$$\widehat{\mathbf{H}}^e_{i(m)} = \widehat{\mathbf{H}}_{i(m)}\widehat{\mathbf{G}}_{i(m)} = \mathbf{W}_{i(m)}\mathbf{H}_{i(m)}(m)\widehat{\mathbf{G}}_{i(m)}. \quad (16)$$

Consequently, the expression given by (13) is reduced to

$$\mathcal{C}^{eBD}_m = \max_{\widehat{\mathbf{P}}} \log \left|\mathbf{I} + \widehat{\mathbf{\Sigma}}^2\widehat{\mathbf{P}}\right|. \quad (17)$$

Notice that, the optimal power loading is in function of $\mathbf{W}_m$. Therefore, each user is required to feedback its whitening matrix to its serving BS.

C. Iterative BD

The BD algorithms presented above have the same dimensionality restrictions. Both expressions given by (6) and (15) have dimension of $[(K-1)N_r \times N_t]$ with rank of $[N_t - (K-1)N_r]$. Therefore, to have d columns in the null space, $[N_t - (K-1)N_r]$ should not be less than d, i.e., $[N_t - (K-1)N_r] \geq d$.

One possible way to reduce the dimensionality restrictions is to use the receive beamforming matrix, $\mathbf{R}_{i(m)}$, when calculating the transmit beamforming, $\mathbf{T}_{i(m)}$[5].

To achieve this end, $\mathbf{R}_{i(m)}$ can be included in (15). Thus, we can write

$$\widetilde{\mathbf{H}}^{-i}_{i(m)} = [\mathbf{R}^H_{1(m)}\widehat{\mathbf{H}}^T_{1(m)} \quad \dots \quad \mathbf{R}^H_{i-1(m)}\widehat{\mathbf{H}}^T_{i-1(m)} \quad \mathbf{R}^H_{i+1(m)}\widehat{\mathbf{H}}^T_{i+1(m)} \quad \dots \quad \mathbf{R}^H_{K(m)}\widehat{\mathbf{H}}^T_{K(m)}]^T. \quad (18)$$

Notice that $\widetilde{\mathbf{H}}^{-i}_{i(m)}$ has dimension of $[(K-1)d \times N_t]$, which is no longer in function of N_r. Calculating the null space from $\widetilde{\mathbf{H}}_{i(m)}$ is always satisfied if, and only if, the number of data streams transmitted by the BS is less than or equal to its number of transmit antennas, i.e., the condition of $[N_t - (K-1)d] \geq d$ should be satisfied.

The following steps are much similar to the ones in the previous sections. The transmit beamforming is given by

$$\widetilde{\mathbf{T}}_{i(m)} = \widetilde{\mathbf{G}}_{i(m)}\widetilde{\mathbf{F}}_{i(m)}\widetilde{\mathbf{P}}^{\frac{1}{2}}_{i(m)}. \quad (19)$$

The $\widetilde{\mathbf{G}}_{i(m)}$ matrix is calculated similar to (8) from $\widetilde{\mathbf{H}}^{-i}_{i(m)}$, that is given by (18). The $\widetilde{\mathbf{F}}_{i(m)}$ and $\widetilde{\mathbf{R}}_{i(m)}$ matrices are calculated using (10) and (11), respectively, from the effective channel of user $i(m)$ that is given by

$$\widetilde{\mathbf{H}}^e_{i(m)} = \widehat{\mathbf{H}}_{i(m)}\widetilde{\mathbf{G}}_{i(m)} = \mathbf{W}_{i(m)}\mathbf{H}_{i(m)}(m)\widetilde{\mathbf{G}}_{i(m)}. \quad (20)$$

Since the transmit and receive beamforming matrices are now coupled, the BS is required to conduct some iterations in order to achieve the BD. Therefore, we refer to this approach as iterative BD (iBD). With the transmit and receive beamforming matrices calculated as above, the expression given by (13) is reduced to

$$\mathcal{C}^{iBD}_m = \max_{\widetilde{\mathbf{P}}} \log \left|\mathbf{I} + \widetilde{\mathbf{\Sigma}}^2\widetilde{\mathbf{P}}\right|. \quad (21)$$

The complete implementation of the iBD at the m^{th} BS is summarized in **Algorithm 1**.

Algorithm 1 iterative BD (iBD).

Initialize: $\widetilde{\mathbf{R}}^{(0)}_{i(m)}, \forall i(m) \in \mathcal{K}_m$.
$k \leftarrow 1$
For user $i(m)$ and $\forall i(m) \in \mathcal{K}_m$ **do**
1) Construct $\widetilde{\mathbf{H}}^{-i(k)}_{i(m)}$ given by (18) using $\widetilde{\mathbf{R}}^{(k)}_{i(m)}$.
2) Calculate $\widetilde{\mathbf{G}}^{(k)}_{i(m)}$ from $\widetilde{\mathbf{H}}^{-i(k)}_{i(m)}$ using (8).
3) Construct $\widetilde{\mathbf{H}}^{e(k)}_{i(m)}$ given by 20 using $\widetilde{\mathbf{G}}^{(k)}_{i(m)}$.
4) Calculate $\widetilde{\mathbf{F}}^{(k+1)}_{i(m)}$ and $\widetilde{\mathbf{R}}^{(k+1)}_{i(m)}$ from $\widetilde{\mathbf{H}}^{e(k)}_{i(m)}$ using (10) and (11).
End.
If $|\mathcal{C}^{iBD}_m(k+1) - \mathcal{C}^{iBD}_m(k)| < \epsilon$, break; otherwise, set $k \leftarrow k+1$.

One important thing to notice from the (16) and (20) expressions is that both have the same structure. The only difference between both is in the null space. Let us assume that user $i(m)$ has N_r receive antennas and receives d data streams. Then, if $d < N_r$, then $\dim(\widehat{\mathbf{G}}_{i(m)}) < \dim(\widetilde{\mathbf{G}}_{i(m)})$; whereas if $d = N_r$, then both have the same dimension. Based on these notations, we have the following theorem.

Theorem 1: If the number of data streams transmitted to any user is equal to the number of its receive antennas, i.e., if $d = N_r$, then eBD and iBD are equivalent and have the same exact performance.

Proof: At first, we notice that the $\widetilde{\mathbf{H}}^{-i}_{i(m)}$ expression given by (18) can be written in function of the $\widehat{\mathbf{H}}^{-i}_{i(m)}$ expression given by (15). To show this, let us define $\widetilde{\mathbf{R}}^{-i}_m = \text{blockdiag}[\widetilde{\mathbf{R}}^H_{1(m)} \dots \widetilde{\mathbf{R}}^H_{i-1(m)} \quad \widetilde{\mathbf{R}}^H_{i+1(m)} \dots \widetilde{\mathbf{R}}^H_{K(m)}]$. Then we can write

$$\widetilde{\mathbf{H}}^{-i}_{i(m)} = \widetilde{\mathbf{R}}^{-i}_m\widehat{\mathbf{H}}^{-i}_{i(m)}.$$

Note that $\widetilde{\mathbf{R}}^{-i}_m$ is an orthogonal unitary matrix. Therefore, if $d = N_r$, then we have $\widetilde{\mathbf{R}}^{H-i}_m\widetilde{\mathbf{R}}^{-i}_m = \mathbf{I}$, otherwise, if $d < N_r$, then $\widetilde{\mathbf{R}}^{H-i}_m\widetilde{\mathbf{R}}^{-i}_m \neq \mathbf{I}$). Assuming $d = N_r$, we have

$$\widetilde{\mathbf{H}}^{H-i}_{i(m)}\widetilde{\mathbf{H}}^{-i}_{i(m)} = \widehat{\mathbf{H}}^{H-i}_{i(m)}\widetilde{\mathbf{R}}^{H-i}_m\widetilde{\mathbf{R}}^{-i}_m\widehat{\mathbf{H}}^{-i}_{i(m)} = \widehat{\mathbf{H}}^{H-i}_{i(m)}\widehat{\mathbf{H}}^{-i}_{i(m)}.$$

Therefore, we have

$$\widetilde{\mathbf{H}}^{-i}_{i(m)} \propto \widehat{\mathbf{H}}^{-i}_{i(m)}.$$

This end result proves that both matrices are proportional to each other. Consequently, their individual null spaces are also proportional to each other, i.e.,

$$\widetilde{\mathbf{G}}_{i(m)} \propto \widehat{\mathbf{G}}_{i(m)}.$$

Thereby, the singular values calculated using (9) assuming $\widetilde{\mathbf{H}}^e_{i(m)}$ and given by (20) are exactly equal to the ones calculated assuming $\widehat{\mathbf{H}}^e_{i(m)}$ given by (16), which completes the proof. ∎

IV. Numerical Results

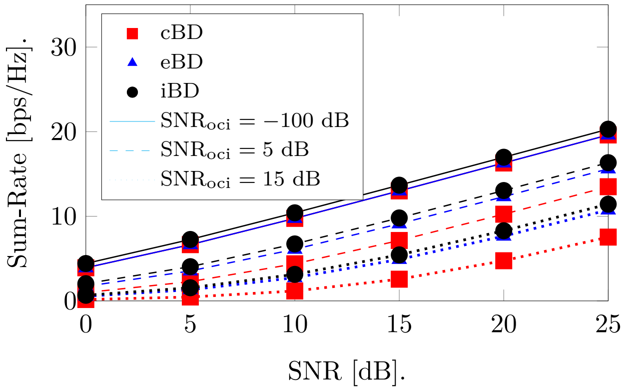
In this section we show simulation results evaluating the BD algorithms presented throughout the paper. We refer to the system setup by $[M, K, N_t, N_r, d]$. The results are an average of 1,000 iterations. For each iteration, new spatially uncorrelated frequency flat fading MIMO channels are generated by complex Gaussian random variables. We assume $\sigma^2_{i(m)} = 1, \forall i(m) \in \mathcal{K}$.

As the system has multiple cells, with eBD and cBD all BSs are required to conduct some iterations starting from initial values. On the other hand, with iBD, each BS is further required to conduct some iterations to achieve the BD between its K users, using Algorithm 1.

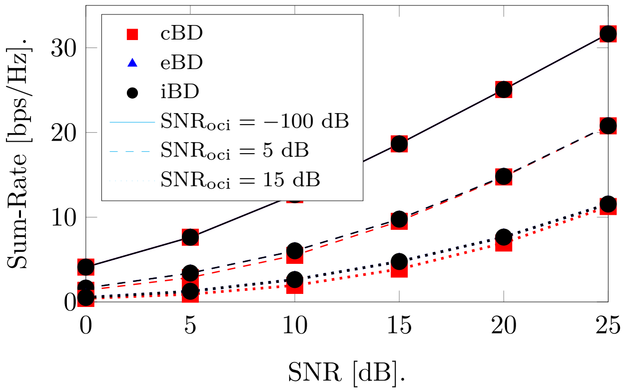
A. Example 1: Single-Cell

In the first example, we assume a system setup of $[3, 2, 6, 2, 1]$. We show the sum-rate results of one cell, say the first cell, as a function of the transmit power of the other 2 cells, denoted by SNR_{oci}. Consequently, the other 2 cells act as the source of the OCI for the first cell.

Figures 2(a) and 2(b) show the sum-rate of the first cell for a range of signal-to-noise ratios (SNRs), assuming each user receives one stream and two streams, respectively.



(a) $d = 1$



(b) $d = 2$

Fig. 2. Example 1: Sum-Rate vs SNR.

From Figure 2(a), we can see that when the other 2 cells increase their transmit power (the OCI increases), the sum-rate of the first cell decreases. Moreover, we can see that iBD outperforms both cBD and eBD for the entire range of SNRs. The iBD algorithm utilizes the rich diversity of the channels by jointly optimizing the transmit and receive beamforming matrices to achieve BD, whereas cBD and eBD depend mainly on the transmit beamforming matrix. As a result, the null space dimension perceived by iBD is larger than the one with cBD or eBD. Additionally, we can see that eBD outperforms cBD, since it accounts for the OCI presence. Notice that, when the $\text{SNR}_{\text{oci}} \to -\infty$ the OCI $\to \mathbf{I}$, thus eBD and cBD have the same performance.

From Figure 2(b), we can see that when the BS is transmitting two data streams ($d = 2$), all algorithms have better sum-rate comparing to results shown in Figure 2(a). Another significant result that we can see is that iBD and eBD have the same exact performance for the entire SNR range. This later result agrees with Theorem 1, which predicts such behavior whenever $d = N_r$.

B. Example 2: Multi-Cell

In the second example, we assume system a setup of $[3, 2, 6, 2, 1]$ and refer to it as the reference system. The number of local users K, transmit antennas N_t, receive antennas N_r and data streams d will be varied while assuming the other parameters are fixed. For all scenarios, we show the sum-rate results of three cells. Figure 3 shows the sum-rate for each scenario for a range of SNRs.

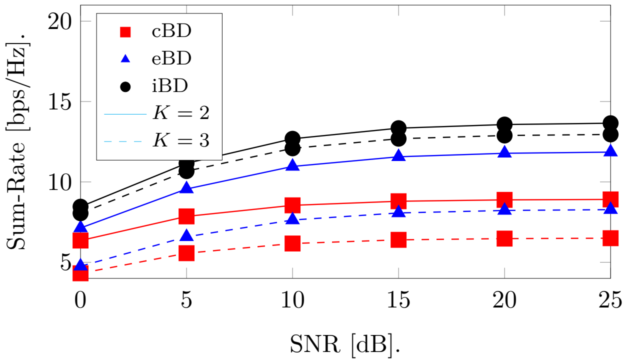
From Figure 3, we can see that all BD approaches start to have a flat performance as the SNR value increases, as a result of the increase of the OCI effects. In Figure 3(a), eBD and cBD are more affected with the increase of the number of local users (K) than iBD. The increase of K reduces the null space dimension for eBD and cBD more significantly than it does for iBD. In Figures 3(b), iBD and eBD both have improved performances with the increase of the number of receive antennas (N_r), while cBD is not affected. The increase of N_r improves the whiting effect for eBD and more significantly for iBD. On the other hand, cBD depends mostly on N_t. Since each BS is sending one stream in both cases, the increase of N_r does not affect the cBD performance. In Figure 3(c), all BD approaches have improved performances with the increase of the number of transmit antennas (N_t), as the dimension of the null space for each approach increases accordingly. In Figure 3(d), all algorithms have degraded performance with the increase of data streams (d), as a results of the increase of the OCI effects. As in the first example, with $d = 2$, both iBD and eBD achieve the same performance.

In terms of convergence, Figure 4 shows the convergence behavior for all algorithms assuming the reference system setup and SNR = 15dB.

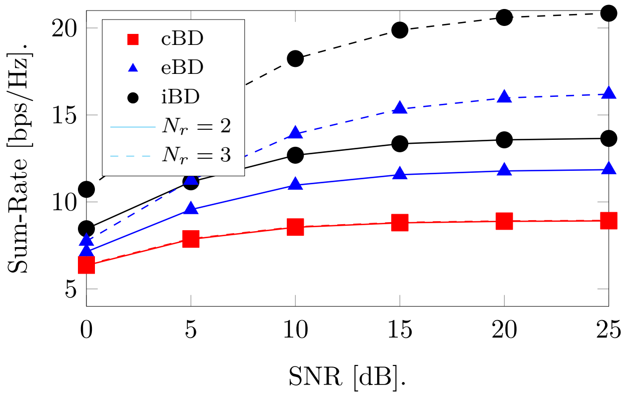
From Figure 4, we can see that all algorithms have a fast convergence, roughly within 5 iterations. These results can be generalized for all scenarios considered in this paper.

V. Conclusions

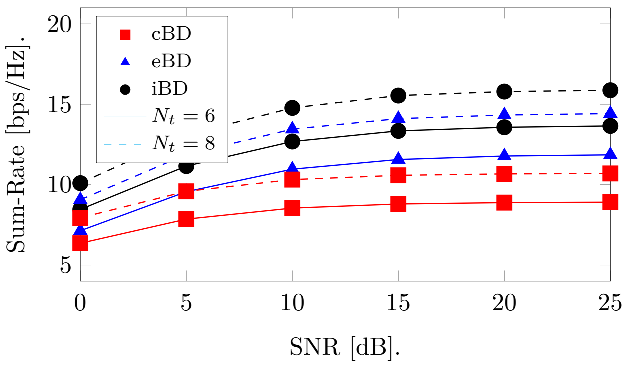
In this paper we have studied the BD approach considering multicell multiuser MIMO systems. The eBD was shown to enhance cBD and reduce the effects of the OCI.



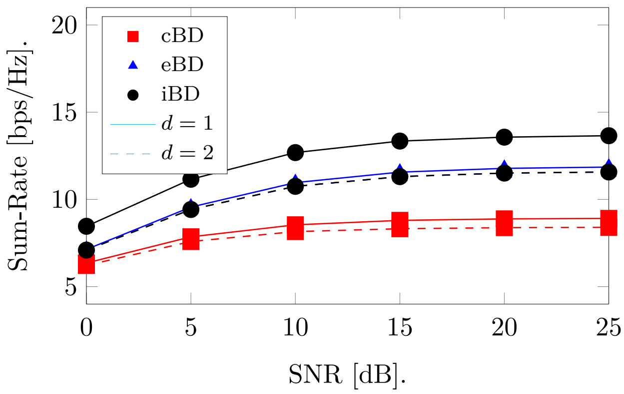
(a) Varying K



(b) Varying N_r



(c) Varying N_t



(d) Varying d

Fig. 3. Example 2: Sum-Rate vs SNR.

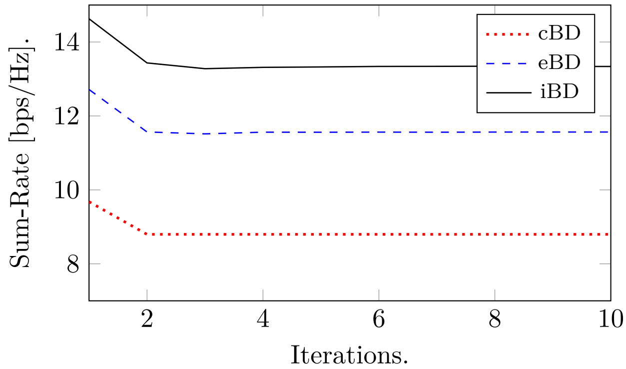


Fig. 4. Sum-Rate vs Iterations.

Although eBD achieves better sum-rate than cBD, it has the same dimensionality restriction. In order to overcome such restriction, the iBD algorithm is proposed in this paper. iBD utilizes the rich diversity of the channels by jointly optimizing the transmit and receive beamforming matrices to achieve BD, while also accounting for the presence of the OCI.

The numerical results verified that iBD outperforms both cBD and eBD. However, the iBD and eBD have the same exact sum-rate if, and only if, the number of the transmitted data streams to a user is equal to its number of receive antennas.

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