New formula proposal for the determination of variable speed pumps efficiency

Proposta de nova fórmula para a determinação do rendimento de bombas de velocidade variável

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ABSTRACT

The use of pumps which work with variable rotation speed consists of an appropriate tactics for reduction of energy costs in the pump systems. In case of change in speed, the pump starts to work in a new flow pattern, head and efficiency values. Two empirical formulas can be found in the literature in order to estimate the final efficiency of a pump in this situation. This article aims at assessing the accuracy of these formulas and, next, to present a new proposal for the efficiency estimate in the new operating conditions. 50 pumps have been chosen and, for each of them, the above-mentioned formulas have been assessed in three procedures of speed reduction. The analysis of these equations' accuracy has been accomplished by means of the comparison between the efficiencies estimated by them with the efficiencies calculated with the aid of the affinity laws. The results indicated that, in 95 of the 150 analysis procedures accomplished, the formula proposed presented estimates which have shown to be more precise than the other two formulas. It has also been possible to verify that the average of error detected through the procedure of the new formula (1.60%) has been lower than the average of the other two (1.93% and 2.22%). In light of this fact, the use of the equation here presented is considered advantageous since, from their more accurate efficiency estimates, the calculations for power and energy are also more precise.

Keywords: Variable speed pumps; Frequency inverter; Efficiency calculation.

RESUMO

O uso de bombas que funcionam com velocidade de rotação variável configura uma tática adequada para que sejam reduzidos os custos de energia nos sistemas de bombeamento. Na ocasião da mudança de velocidade, a bomba passa a trabalhar em novo regime de vazão, carga manométrica e valores de rendimento. Podem ser encontradas na literatura duas fórmulas empíricas para se estimar o rendimento final de uma bomba nessa situação. Este artigo objetiva avaliar a precisão dessas fórmulas e, em seguida, apresentar uma nova proposta para a estimativa do rendimento nas novas condições de operação. Escolheram-se 50 bombas e, para cada uma, as fórmulas mencionadas foram avaliadas em três procedimentos de redução de velocidade. A análise da precisão dessas equações foi realizada através da comparação dos rendimentos estimados por elas com os rendimentos calculados com o auxílio das leis de semelhança. Os resultados apontaram que, em 95 dos 150 procedimentos de análise realizados, a fórmula proposta apresentou estimativas mais precisas do que as outras duas fórmulas. Verificou-se, ainda, que a média de erro por procedimento da nova fórmula (1,60%) foi menor do que as médias das outras (1,93% e 2,22%). Em vista disso, a utilização da equação ora apresentada é considerada vantajosa uma vez que, a partir de suas estimativas mais acuradas de rendimento, os cálculos de potência e energia serão também mais precisos.

Palavras-chave: Bombas de velocidade variável; Inversor de frequência; Cálculo de rendimento.

INTRODUCTION

The operation of water supply networks is a task which commonly requires the execution of hydraulic pumps, whose function is to enable the water to be conducted in appropriate amount and pressure to its users. According to the accounts of Sârbu and Borza (1998), in order to guarantee an ideal pumping pressure, a proportion between 70 and 80% of all the energy consumed in many water distribution systems is exclusively destined to this purpose.

Due to the significative amount of energy used in the pumping, the work of conducting the water with maximum efficiency is treated as an indispensable requirement in several areas, such as in sanitation, for irrigation purposes and also in many other applications which utilize hydraulic pumps, since they reduce the costs concerning the assembly of the pumping system, as well as its maintenance and the energy expenses. In order to do so, the accomplishment of laboratory experiments which aim at analyzing the operation conditions of a hydraulic pump and its efficiency is common (CIPOLLA et al., 2011).

The costs related to the working of the pumps can be reduced by minimizing energy consumption. An interesting strategy to fulfill this goal is the utilization of pumps whose rotation speed is variable instead of those whose rotation speed is steady (MARCHI; SIMPSON; ERTUGRUL, 2012). The variable speed pumps are connected to a motor which is controlled by a frequency inverter, whose function is to modify the power supply in order to change the pump's rotation, making it work under the maximum efficiency possible.

Commonly, the majority of the industrial applications is based on electronic instruments which work by means of speed change. Among the main characteristics of triggering by variable speed, it is possible to highlight simplicity, triviality in its handling and the possibility of operation in high power values (SOUSA; SILVA; PIRES, 2012).

By handling the motor's rotation speed, which is proportioned by the frequency inverter, the pump's efficiency curve is modified and the installation curve is unaltered. Thus, the energy consumed is directly proportional to the rotation and, therefore, it is precisely the amount necessary, neither lacking nor exceeding the energy consumption (RODRIGUES, 2007).

The operation point of a centrifugal pump (defined by a pair of flow and head values) is usually controlled by adjusting itself to the pump's rotation speed, aiming at the reduction in its energy consumption. In general, the speed control is accomplished with a frequency converter which enables the control of an induction motor's rotation speed.

The utilization of variable speed pumps can promote a significant economy in energy if the supply system accepts a high variability of operating conditions, so that the benefits of adjusting it to its operation point can be widely explored. Particularly, the decrease in energy consumption comprises the hypothesis of reducing itself to height or flow in the system.

In hot water systems, the possibility of variation in the pump's rotation speed by using an inverter enables to adjust the pressures to the instantaneous thermal load. Besides directly influencing efficiency, the speed reduction entails other advantages, such as lower bearing loads, higher degree of reliability, lower maintenance costs and reduced rates of fugitive emissions, discarding the control valve from the pipe (SÂRBU; VALEA, 2015).

In many cases, due to the system's variable demand, it is verified that the pumps operate with an efficiency which is considerably lower than the one that would be possible, impacting on energy consumption. The increase of public interest in energy efficiency has turned optimization in the energy consumption of pumping systems a widely studied theme (GEORGESCU et al., 2014).

In the year of 2013, an efficiency analysis research on pumps was accomplished in eight cities in the province of Ontario, in Canada. In this research, it has been verified that the majority of these pumps works with efficiencies which are distant form their maximum values. As a consequence of this inefficacy, there is an annual waste which amounts to approximately 33.7GWh (HYDRATEK, 2013).

Due to the fact that the change in rotation leads to the slide of the pump's operating point, causing alterations in the flow measures, head, power and efficiency, it becomes necessary to make an estimate of the new efficiency under which the pump will operate in its new characteristics with the maximum accuracy, since it is directly connected to the energy consumption of the pumping system.

Furthermore, the energy consumed by a pump in a system is given by the product between the water specific weight (γ), the pumping flow (Q), the head (H) and the time (t), divided by the efficiency of the motor-pump set. Thus, an inaccurate estimate of the efficiency might imply in an underestimate or overestimate of the energy consumption, depending on the characteristics of the system.

In order to accomplish efficiency estimates in the new conditions of the pump's operation, there are two empirical formulas in the literature concerning the subject. The first formula is Equation 1 and it has been proposed by Raymond Comolet (1961).

$$\eta_2 = \frac{\eta_1}{\eta_1 + (1 - \eta_1).R^{-0.17}} \tag{1}$$

The second formula is Equation 2 and it has been proposed by Sârbu and Borza (1998).

$$\eta_2 = 1 - (1 - \eta_1).R^{-0,1} \tag{2}$$

In both equations, R is the relation of division between the rotation values (N_2/N_1), η is the efficiency and indexes 1 and 2 are related to the initial and final states, respectively.

In light of its importance, the goal of this article is to develop a formula to calculate the new efficiency of a variable speed pump when its operating point is altered and, then, to compare the results of the formula here proposed with the results of the formulas proposed by Raymond Comolet (1961) and Sârbu and Borza (1998).

CHANGE IN ROTATING SPEED

Equations 3, 4 and 5 represent the dimensionless numbers, which serve to model the behavior of two similar pumps in the occasion of change in rotating speed (N) or in the diameter (D) with

all the other dimensions of the impeller (MARCHI; SIMPSON, 2013). The magnitudes C_Q, C_H e C_P are denominated, respectively, flow coefficients, head and power.

$$C_Q = \frac{Q}{N.D^3} \tag{3}$$

$$C_H = \frac{g.H}{N^2.D^2} \tag{4}$$

$$C_P = \frac{P}{\rho.N^3.D^5} \tag{5}$$

It is worth highlighting that Q is the flow, H is the head and P is the power, ρ is the liquid's specific mass and g is the gravitational acceleration.

The above-mentioned dimensionless numbers have constant values when two pumps are geometrically similar. Therefore, if there is any alteration in the rotation speed or in the diameter of a pump, its characteristics can be represented in graphs only by means of the curves C_H x C_Q and η x C_Q (COUTINHO, 2015).

Considering the existence of two similar pumps, with identical dimensions, conducting the same fluid, working in different speeds (N_1 and N_2), it is also possible to consider that the dimensionless numbers are maintained as constant and, therefore, eliminating the invariant magnitudes ρ, g and D, leading to the deduction of Equations 6, 7 and 8, called affinity laws (SANTOS, 2007).

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \tag{6}$$

$$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \tag{7}$$

$$\frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^3 \tag{8}$$

Two ordered pairs (Q_1, H_1) and (Q_2, H_2) which satisfy the affinity laws are called homologous points (HP) and necessarily have equal efficiencies.

$$\eta_1 = \eta_2 \tag{9}$$

Equations 6 and 7 provide a good approximation of a centrifugal pump's behavior for a wide speed range, and the impact of the affected factors by its size can be easily discarded. On the other hand, the approximation in the relations of power and efficiency is higher, especially for the smaller pumps (SIMPSON; MARCHI, 2013).

Equation 10 represents the generic performance curve of a pump, which shows how the head varies according to the flow's variation.

$$H = aQ^2 + bQ + c \tag{10}$$

It is worth highlighting that the constants a, b and c are inherent to each pump.

In order to simulate the pump's behavior when the change in its speed occurs, it is necessary to determine the equation of its performance curve in the case of its operation to the nominal rotation speed. The pump's performance curves equations for other speed values can be determined with the support of the affinity laws (MESQUITA et al., 2006).

Considering the fact that a pump which operates in a given rotation is similar to itself operating the other rotation, a given point of its performance curve in speed N_1 (Q_1, H_1) admits another point (Q_2, H_2), which is homologous to the first one, in the performance curve related to speed N_2, and which are associated by means of the affinity laws.

Based on Equations 6 and 7, it is shown that, for any other speed N_2, the pump has as its performance curve equation the following:

$$H_2 = aQ_2^2 + b\frac{N_2}{N_1}Q_2 + c\left(\frac{N_2}{N_1}\right)^2 \tag{11}$$

The system curve represents the head variation which is necessary to the installation as an expression of the fluid's boosting flow. Considering that the term related to head loss is based on the universal formulation, the equation of this curve is given by:

$$H = H_g + mQ^2 \tag{12}$$

It is worth highlighting that H_g is called geometrical height of the installation and m is a term concerning the head loss, which is a function of other magnitudes in example of the gravitational acceleration, the pipe's diameter and the friction factor.

The pump's curve (PC) and the system curve (SC) are disposed at Figure 1. The pump's curve represents the energy provided to the fluid whereas the system curve represents the energy the fluid must have to flow in the geometrical and hydraulic conditions of the system. Therefore, by joining necessity and availability, their intersection point appears, known as the pump's operating point (OP) (SANTOS, 2007).

With a decrease in speed, the pump's curve is altered and, therefore, its operating point is displaced. However, as it is shown in figure 2, the points of initial (Q_i, H_i) and final (Q_f, H_f) operation will not necessarily meet the affinity laws, in other words, they will not necessarily have equal efficiencies. The point (Q_f, H_f) is homologous to (Q'_f, H'_f) and not to (Q_i, H_i). Thus, it is necessary to determine the pump's efficiency in its new operating conditions.
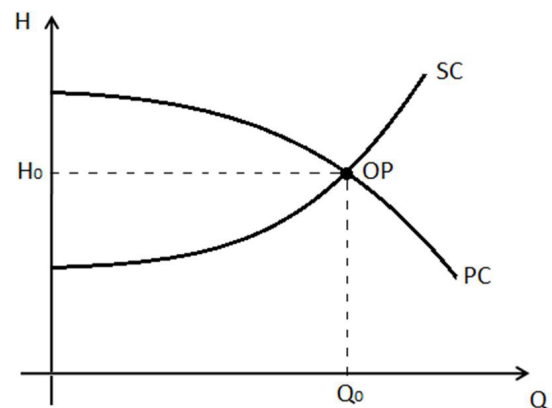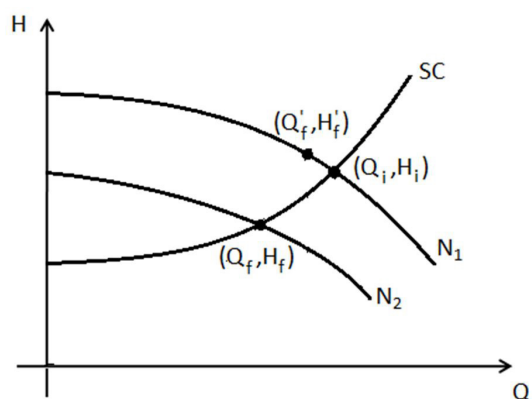

Figure 1. A pump's operating point.

Figure 2. Operating points in two rotation speeds N_1 and N_2 and a point which is homologous to the final operating point.

Figure 3. Operating points of a pump used for the above-mentioned rotation speeds.

METHODOLOGY

The equations of efficiency estimates existent in the literature (Comolet and Sârbu and Borza) have been assessed by means of procedures reported as follows. According to the results provided by the above-mentioned procedures, it is also possible to adjust the parameters of the analytical formula which has been initially deducted during the execution of this research, which makes the final efficiency values more accurate.

In order to determine the coordinates of each pump's operating point and calculate the efficiencies in their respective points, a computer program in the learning environment Free Pascal Lazarus has been developed. Besides the execution of the above-mentioned calculus routine, it was possible to adjust parameters of the analytical formula proposed by means of the software developed, making the final efficiency values estimated by it more accurate.

Firstly, 5 distinct brands of pumps have been chosen and, next, 10 pumps of each one of them, totalizing 50 pumps. Based on the information of the manuals provided by the manufacturers, the pump's curve and the efficiency curve concerning the normal speed of each of the referred pumps have been obtained. It is worth mentioning that in the catalogs of all the pumps used, the flow values are given in cubic meters per hour (m^3/h) and the head is given in meters.

After obtaining the equation of the pump's curve at the nominal rotation, it was possible to determine the respective equations for other three rotation values (90%, 80% and 70% of the nominal value). The choice of these three values is due to the fact that, for speed reductions which are higher than 1/3 of the nominal rotation, the affinity laws present substantial errors. Thus, in each pump taken, the equations proposed by Comolet and by Sârbu and Borza have been assessed three times, meaning 150 analysis procedures for both.

In order to determine the operating points, a distinct system curve has been attributed to each of the pumps, so that it could cross the four pumps' curves obtained so far. The four points here determined have been designed according to Figure 3, which presents such information for one of the fifty pumps chosen.
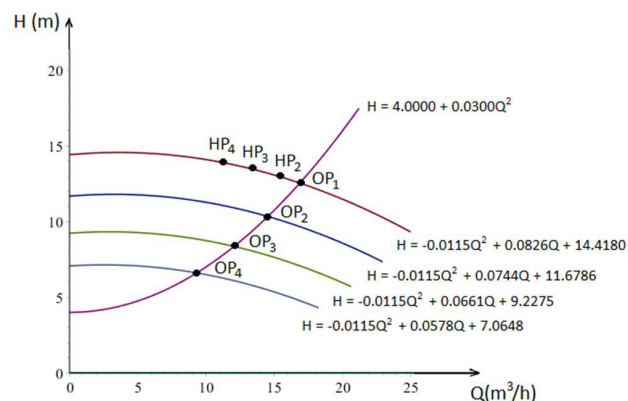
Table 1 presents the values of coefficients a, b and c of the equation for the performance curve at nominal speed, coefficients e, f and j of the equation for the efficiency curve and H_g and m of the equation attributed to the system curve for each pump utilized. The equations of the curves which are characteristic of the pump (performance and efficiency) have been obtained from the graphs contained in their catalogs and with the aid of Microsoft Excel, disposing the equations of the above-mentioned curves and the correlation coefficient r^2 among the variables involved.

Equation 13 conveys the mathematical expression of the efficiency curve concerning the nominal speed. The obtention of this equation for each pump has been accomplished by non-linear regression from the graph of its own curve or from ordered pairs (Q, η) provided in the manufacturer's manual.

$$\eta = eQ^2 + fQ + j \qquad (13)$$

It is worth highlighting that the coefficients e, f and j are characteristic of each pump.

It is possible to notice that point OP_1 belongs to the pump's curve referent to the nominal rotation. Thus, in order to calculate the efficiency η_1 in this point, its flow value in the equation of the efficiency curve must be replaced.

When it comes to the other points (OP_2, OP_3 and OP_4), they do not belong to the pump's curve related to the nominal rotation and, for this reason, its flow values must not be replaced in the efficiency equation. It is necessary, however, to utilize the affinity laws to determine the points of the above-mentioned curve which are homologous to the referred points. The points found through this way will be denominated HP_2, HP_3 and HP_4. So, the flow values of the news points must be replaced in the efficiency curve's equation and, then, the efficiencies in OP_2, OP_3 and OP_4 will be determined.

Finally, the efficiencies of OP_2, OP_3 and OP_4 have been estimated through the equations proposed by Comolet and by Sârbu and Borza and through the equation proposed in this paper. These efficiency estimates have been compared with the values calculated by the affinity laws, finding the difference between estimated efficiencies and calculated efficiency. Finally, for each one of the pumps and for each analysis procedure accomplished,

Table 1. Data concerning the curves which are characteristic of the pumps utilized.

Pump	Pump's Curve				Efficiency Curve				System Curve	
	a	b	c	r²	e	f	j	r²	Hg	m
P1	-0.000073	0.007172	96.788	**0.999**	-0.00000211	0.002548	0.0989	**0.999**	20	0.0003
P2	-0.01148	0.08262	14.418	**0.997**	-0.0014	0.05066	0.175	**0.997**	4	0.03
P3	-0.000164	0.01397	14.467	**0.996**	-0.00002408	0.008692	0.04387	**0.999**	3	0.0003
P4	-0.000029	0.01122	47.997	**0.990**	-0.000001735	0.002369	0.057583	**0.995**	10	0.0001
P5	-0.0117	0.05943	6.292	**0.997**	-0.00323	0.07702	0.1743	**0.997**	2	0.02
P6	-0.001113	0.009422	4.07	**0.996**	-0.00065	0.0407	0.157	**0.991**	1	0.002
P7	-0.000442	0.01827	10.516	**0.992**	-0.0001238	0.01941	0.08683	**0.996**	2	0.001
P8	-0.00023	0.008235	25.882	**0.997**	-0.00002245	0.006834	0.29303	**0.989**	5	0.001
P9	-0.000036	0.002566	9.253	**0.999**	-0.00000823	0.004891	0.08346	**0.997**	2	0.0001
P10	-0.000029	0.00751	21.09	**0.991**	-0.00000391	0.003551	0.06021	**0.995**	4	0.0001
P11	-0.02385	0.3135	58.65	**0.994**	-0.0007429	0.03994	0.06167	**0.974**	20	0.07
P12	-0.00886	0.2923	87.637	**0.986**	-0.0002281	0.02338	0.07359	**0.998**	25	0.02
P13	-0.001198	0.08503	132.98	**0.989**	-0.00002796	0.008136	0.16323	**0.988**	40	0.005
P14	-0.008035	0.1235	22.01	**0.993**	-0.0008249	0.04456	-0.02139	**0.998**	5	0.02
P15	-0.003654	0.06892	34.855	**0.998**	-0.0002851	0.02633	0.02772	**0.997**	10	0.015
P16	-0.000608	0.0252	8.818	**0.987**	-0.0001679	0.02328	-0.0322	**0.998**	2	0.002
P17	-0.001039	0.03494	24.286	**0.993**	-0.0001175	0.01682	0.18884	**0.998**	6	0.004
P18	-0.001354	0.04887	31.415	**0.991**	-0.0001078	0.01588	0.09462	**0.995**	8	0.005
P19	-0.000256	0.02148	34.679	**0.996**	-0.00002165	0.007944	0.11272	**0.987**	10	0.001
P20	-0.000059	0.01496	51.888	**0.995**	-0.00000288	0.00293	0.12297	**0.994**	15	0.0002
P21	-0.3101	-1.337	35.296	**0.999**	-1.841	15.603	1.295	**0.999**	10	0.9
P22	-0.139	-0.8091	20.478	**1**	-1.502	13.833	1.917	**0.999**	5	0.4
P23	-0.01366	0.07833	21.183	**1**	-0.1386	5.838	1.17	**0.998**	4	0.03
P24	-0.01471	0.137	26.339	**1**	-0.1079	5.045	1.867	**1**	5	0.03
P25	-0.000875	0.02407	51.754	**0.999**	-0.003314	0.9466	-0.004766	**1**	15	0.003
P26	-0.02222	0.01428	34.743	**0.998**	-0.3313	9.098	-0.1214	**1**	8	0.25
P27	-0.2276	-0.3125	51	**1**	-0.5131	7.696	-0.5261	**0.997**	10	0.5
P28	-0.5571	-0.699	157.97	**0.999**	-0.4819	7.944	0.006646	**1**	40	0.9
P29	-0.1706	-0.276	5.991	**1**	-5.313	28.077	1.035	**0.999**	1	0.4
P30	-0.01666	-0.2732	9.104	**1**	-0.4744	8.246	1.033	**1**	2	0.08
P31	-0.0291	-0.2628	29.84	**0.998**	-0.1781	5.21	0.6554	**0.987**	7	0.06
P32	-0.03284	0.07643	11.412	**0.996**	-0.7855	10.216	7.637	**0.957**	2	0.15
P33	-0.009321	0.02675	11.152	**1**	-0.2167	7.061	-5.264	**0.994**	2	0.02
P34	-0.01879	0.2267	83.751	**0.993**	-0.04998	3.434	-11.35	**0.988**	20	0.04
P35	-0.003859	0.05541	37.276	**0.999**	-0.0168	1.703	3.303	**0.996**	10	0.008
P36	-0.001343	0.01734	10.68	**0.999**	-0.03104	2.737	4.115	**0.991**	2	0.004
P37	-0.001149	-0.01123	76.828	**0.999**	-0.004187	1.026	5.708	**0.998**	20	0.005
P38	-0.001374	0.01643	44.996	**0.997**	-0.006767	1.262	-8.649	**0.986**	10	0.004
P39	-0.000186	0.03557	31.232	**0.994**	-0.00159	0.719	-9.642	**0.970**	8	0.0005
P40	-0.000059	-0.002235	23.357	**0.999**	-0.000375	0.2304	16.675	**0.987**	5	0.0001
P41	-0.166	0.4487	23.226	**0.998**	-1.204	15.801	-6.101	**0.996**	5	0.4
P42	-0.1005	0.1486	11.554	**0.999**	-1.559	17.763	3.246	**0.995**	2	0.15
P43	-0.07465	0.1058	32.043	**1**	-0.5367	11.031	2.304	**0.992**	7	0.2
P44	-0.02102	0.08992	10.923	**0.999**	-0.3907	9.933	1.098	**0.996**	2	0.05
P45	-0.04635	0.006302	29.767	**0.998**	-0.3274	8.108	4.679	**0.994**	7	0.1
P46	-0.01639	0.1752	59.99	**0.999**	-0.04875	3.126	4.423	**0.991**	15	0.03
P47	-0.1607	-0.7119	19.304	**0.998**	-2.591	24.06	-0.5541	**0.998**	5	0.4
P48	-0.000075	0.01855	33.294	**0.997**	-0.000447	0.3839	0.02657	**0.999**	8	0.0002
P49	-0.001148	0.02967	49.465	**0.999**	-0.005551	1.234	0.9145	**0.991**	12	0.002
P50	-0.002095	0.085	20.068	**0.999**	-0.01474	2.012	-1.294	**0.999**	5	0.005

it was possible to observe which among these three equations enabled a more accurate efficiency estimate.

Through the values of the errors in the equations analyzed, it was possible to interpret, in each test, which of them provides estimates of final efficiencies which are closer to those calculated with the aid of the affinity laws. The error concerning the estimate of each equation has been defined by means of the following equation:

$$Erro = \frac{\eta_{estimated} - \eta_{calculated}}{\eta_{calculated}} \times 100\% \tag{14}$$

in which $\eta_{estimated}$ is the efficiency which has been estimated by any of the formulas analyzed (Sârbu and Borza, Comolet and formula proposed) and $\eta_{calculated}$ is the efficiency calculated with the aid of the affinity laws.

NEW FORMULA PROPOSAL FOR THE DETERMINATION OF EFFICIENCY

Besides analyzing the accuracy of the equations mentioned in the literature, one of the tasks this study proposes to perform is to develop a new formula for the calculation of a pump's final efficiency, when the operating point is displaced due to the variation in rotation speed. The following demonstration, proposed in this research, is based on the affinity laws, which rules the behavior of homologous points, and in the quadratic form of the equations concerning the pump's characteristic curves (performance curve and efficiency curve).

At Figure 4, the pump's performance curves for both speeds in distinct rotations are N_1 and N_2 and the system curve, which intersects the previous ones in the operating points X and Y. Being point Z homologous to Y in the curve related to rotation N_1, the thinnest curve which goes through Y and Z is called Affinity Curve (AC). Q_x, Q_y and Q_z are the flows of points X, Y and Z, respectively.

According to what has been previously mentioned, Equations 10 and 11 model the efficiency curves according to the respective rotation speeds N_1 and N_2 and Equation 13 rules the pump's efficiency for the nominal speed.

The pump's efficiency in the operating point X is given by:

$$\eta_1 = eQ_x^2 + fQ_x + j \tag{15}$$

Since Y and Z are homologous, it is possible to affirm that the pump's efficiency in the point Y is given by:

$$\eta_2 = eQ_z^2 + fQ_z + j \tag{16}$$

From the affinity laws, it is possible to affirm that:

$$Q_y = Q_z \frac{N_2}{N_1} \tag{17}$$

In the graph, it is possible to observe that Q_z is a value which is necessarily located between Q_x and Q_y. Thus, it is possible to assure that Q_z is given by a weighted average between Q_x and Q_y. In other words, it is possible to assert that:

$$Q_z = p \cdot Q_x + (1 - p) \cdot Q_y \tag{18}$$
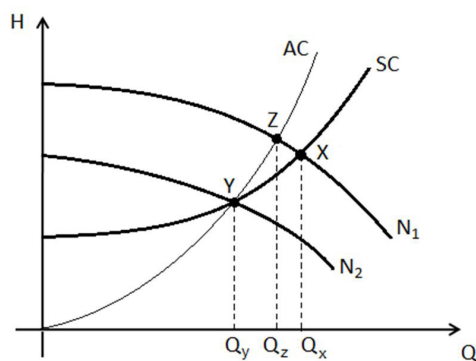
Figure 4. Performance curves for distinct speeds N_1 and N_2, system curve and affinity curve.

where p is a value between 0 and 1.

By hypothesis, it is admitted that p = 0.5 or, equivalently, that Q_z is the arithmetic means between Q_x and Q_y. This hypothesis will be more reliable the lower the variation in rotation speed is. Therefore, we have, as follows:

$$Q_z = \frac{Q_x + Q_y}{2} \tag{19}$$

By replacing in Equation 19 the value of Q_y isolated in Equation 17, we have:

$$Q_z = \frac{Q_x + Q_z \frac{N_2}{N_1}}{2} \tag{20}$$

By rearranging it and isolating Q_z in function of Q_x, we have:

$$Q_z = \frac{Q_x}{\left(2 - \frac{N_2}{N_1}\right)} \tag{21}$$

Subtracting Equations 16 and 15, we have:

$$\eta_2 - \eta_1 = e\left(Q_z^2 - Q_x^2\right) + f\left(Q_z - Q_x\right) \tag{22}$$

Or, also:

$$\eta_2 - \eta_1 = \left(Q_z - Q_x\right)\left[e\left(Q_z + Q_x\right) + f\right] \tag{23}$$

By replacing in Equation 23 the value of Q_z isolated in Equation 21, we obtain:

$$\eta_2 - \eta_1 = \left(\frac{N_2 - N_1}{2N_1 - N_2}\right)\left[eQ_x^2\left(1 + \frac{N_1}{2N_1 - N_2}\right) + fQ_x\right] \tag{24}$$

Or, correspondently, to:

$$\eta_2 - \eta_1 = \left(\frac{N_2 - N_1}{2N_1 - N_2}\right)\left[eQ_x^2 + fQ_x + eQ_x^2 \frac{N_1}{2N_1 - N_2}\right] \tag{25}$$

By hypothesis, the following approximation is possible:

$$\eta_1 \cong eQ_x^2 + fQ_x \tag{26}$$

This hypothesis is considered as positive with relation to η_1 calculated by Equation 15, since in the equations of the pumps' efficiency curves, the independent coefficient value f is significantly close to zero. Considering j = 0 mathematically means to affirm that the pump's efficiency is null for a null rotation, which is coherent with its operation. Therefore, we obtain the following expression:

$$\eta_2 - \eta_1 = \left(\frac{N_2 - N_1}{2N_1 - N_2}\right)\left[\eta_1 + eQ_x^2 \frac{N_1}{2N_1 - N_2}\right] \tag{27}$$

By hypothesis, the following approximation is possible:

$$\eta_1 \cong eQ_x^2 \tag{28}$$

This last hypothesis is proposed with the purpose of evidencing the initial efficiency η_1 in the second member of Equation 27. The errors resulting from it will be assessed afterwards. Thus, we have:

$$\eta_2 - \eta_1 = \left(\frac{N_2 - N_1}{2N_1 - N_2}\right)\left[\eta_1 + \eta_1\frac{N_1}{2N_1 - N_2}\right] \qquad (29)$$

Rearranging the terms, we have:

$$\eta_2 - \eta_1 = \left(\frac{N_1}{2N_1 - N_2} - 1\right)\eta_1\left(1 + \frac{N_1}{2N_1 - N_2}\right) \qquad (30)$$

$$\eta_2 = \eta_1 + \eta_1\left[\left(\frac{N_1}{2N_1 - N_2}\right)^2 - 1\right] \qquad (31)$$

$$\eta_2 = \eta_1\left(\frac{N_1}{2N_1 - N_2}\right)^2 \qquad (32)$$

R being the ratio between the rotation speeds N_2/N_1, also denominated relative rotation speed, finally gets to:

$$\eta_2 = \frac{\eta_1}{(2 - R)^2} \qquad (33)$$

Equation 33 has been assessed, together with the formulas proposed by Comolet and Sârbu and Borza, in the 150 analysis procedures accomplished. The results have shown that the efficiency values provided by it presented errors which were way higher than the errors of the referred empirical formulas existent in the literature.

However, with the intention to improve the accuracy of the efficiency values determined by Equation 33, the following alterations were proposed:

$$\eta_2 = \frac{k.\eta_1}{(2 - R)^w} \qquad (34)$$

The modifications consist of inserting parameters k and w. The adjustment of these parameters has been accomplished by means of the computer tool which has been developed and will be described afterwards.

During such procedure, a hypothesis raised is that w would not be a steady number, but an expression of related speed rotation R. Some functions of different natures (polynomial, sinusoidal, radical and logarithmical) have been tested in place of w. The expressions which adapted the efficiency values estimated by the formula proposed to the values calculated by the affinity laws were the functions which involved R's natural logarithm. So, the following equation has been proposed for w:

$$w = \alpha + \beta.\ln R \qquad (35)$$

in which α and β are real numbers.

With the aid of the software developed, by altering the values for k, α and β in its source code, the ideal values k =1, α = 0 and β = - 0.4 have been found. Finally, it was possible to obtain the following formula:

$$\eta_2 = \eta_1.(2 - R)^{[0.4\ln R]} \qquad (36)$$

By means of the above-mentioned program, it was possible to measure the accuracy of the efficiency values determined by Equation 36, comparing them to those estimated by the formulas proposed by Comolet and Sârbu and Borza.

COMPUTATIONAL TOOL

With the purpose of performing the calculation related to the tests in a systematic and efficient way, a computational program in the development environment Free Pascal Lazarus has been constructed. The program utilizes the pump's curve equation and the efficiency curve equation to the nominal speed as entry dates.

After being executed, the referred computational program developed exhibits the following information for each test accomplished as a report:

- Pump's initial efficiency (at nominal speed);

- Pump's Final Efficiency (at reduced speed) calculated with the aid of the affinity laws, or simply called "calculated efficiency";

- The pump's final efficiency (at reduced speed) estimated by Comolet's Formula;

- The pump's final efficiency (at reduced speed) estimated by Sârbu and Borza's Formula;

- The pump's final efficiency (at reduced speed) estimated by the formula proposed;

- Error (%) between the efficiency estimated by the Comolet Formula and the efficiency calculated;

- Error (%) between the efficiency estimated by Sârbu and Borza's formula and the efficiency calculated;

- Error (%) between the efficiency estimated by the formula proposed and the efficiency calculated;

- Total sum and average of the errors' absolute values (%) concerning each of the empirical formulas mentioned and the formula proposed.

Then, the software has enabled the adjustment of parameters k and w of Equation 34. Their estimate has been accomplished by having in mind the minimization of the sum and also of the average of the errors' absolute values of the formula proposed in the 150 procedures.

RESULTS

The results of this work are presented in two stages. Firstly, the comparison between the formula proposed and the formulas proposed by Comolet and also by Sârbu and Borza has been made by means of average statistics concerning the errors related to each one of them. Next, a quantitative survey of the procedures in which the formula proposed has provided efficiency estimates which have shown to be more accurate than the other two formulas.

Comparison between the absolute errors of the empirical formulas and of the formula proposed

The software created has generated a table with the efficiency values calculated in the four operating points for each pump (OP$_1$, OP$_2$, OP$_3$ and OP$_4$), the efficiency values estimated in the points OP$_2$, OP$_3$ and OP$_4$ by the formulas analyzed and, also, the errors concerning each one of them. For instance, Table 2, which shows this information concerning pump 1. It is worth highlighting that the error of each estimate increases as far as the speed decreases, a fact which has been observed for all the pumps used.

The data informed in the 50 tables has been synthetized so that the errors of the three formulas in each analysis procedure have been transformed into absolute values. Therefore, the sum and the average of the absolute errors concerning the three equations analyzed in the 150 procedures have been calculated and, next, such information has been disposed at Table 3.

It is possible to observe that Sârbu and Borza have presented the highest sum values and errors average, in absolute values. Comolet's formula, on its turn, has shown to be a bit more precise than Sârbu and Borza's formula, with 1.93% of average error per procedure accomplished, whereas the formula here proposed has shown itself to be more accurate than the other two, having presented an average error rate of 1.60% by procedure.

Having in mind that the errors in the efficiency estimates become higher whereas the pump's rotation decrease, an analysis of these errors has been accomplished in three distinct scenarios, according to the percentage of speed reduction with relation to the nominal value. The total number of procedures accomplished (150) has been subdivided in three parts of equal sizes. The first part, or Scenario 1, refers to the 50 procedures in which the speed of each pump has been reduced in 10% of its nominal value. In the occasions when the pump's rotation has been reduced in

20% and 30 % of the nominal value, we have the procedures which compose Scenarios 2 and 3, respectively.

Table 4 has also been constructed from data synthetized in the tables generated by the above-mentioned software. At the same table, the mean absolute errors of each formula per scenario considered is presented.

According to the data observed, the equation proposed by Sârbu and Borza has provided a lower degree of accuracy in the three scenarios, whereas the equation proposed by Comolet has shown to be a bit more accurate than the former. With relation to the equation proposed in this work, its level of accuracy has been verified to be better than the previous ones thanks to its mean absolute error per procedure, which is the lowest.

Advantageous procedures of the formula proposed with relation to the empirical formulas

The accuracy level of the formula here proposed can also be compared to the level of the formulas proposed by Comolet and also by Sârbu and Borza from another evaluative behavior. A survey on the amount of procedures in which the new formula has shown to be more accurate than any of the other two formulas has been accomplished.

The absolute values of the errors related to each procedure accomplished have been printed in graphs so that each formula analyzed is represented by a specific symbol. The efficiency estimate of each equation is as precise as the closer its representative symbol is to its horizontal axis.

In the graphs disposed from Figures 5 to 7, the procedures executed have been numbered from 1 to 150. It is worth highlighting that in pump P1, the procedures of number 1, 2 and 3 had been accomplished; at pump P2 the procedures identified as 4, 5 and 6; and, generically speaking, in the pump P$_i$ (i being an integer number from 1 to 50), the procedures numbered by 3i − 2, 3i − 1 and 3i.

Table 2. Summary of the tests at pump 1 (Brand A) – nominal speed: 3500rpm.

Operation Point (Rotation Speed)	η Calculated (%) Affinity Laws	η Estimated (%)			Error (%)		
		Comolet	S. and Borza	Proposal	Comolet	S. e Borza	Proposal
OP$_1$ (3500 rpm)	82.66	---	---	---	---	---	---
OP$_2$ (3150 rpm)	81.78	82.40	82.47	**82.33**	0.75	0.85	**0.67**
OP$_3$ (2800 rpm)	80.36	82.11	82.27	**81.32**	2.17	2.37	**1.20**
OP$_4$ (2450 rpm)	77.80	81.77	82.03	**79.62**	5.10	5.43	**2.34**

Table 3. Sum and average of the formulas' absolute errors.

	Absolute Errors (%)		
	Comolet	Sârbu	Proposal
Sum	289.92	333.07	**240.07**
Average	1.93	2.22	**1.60**

Table 4. Mean absolute errors of the formulas per scenario.

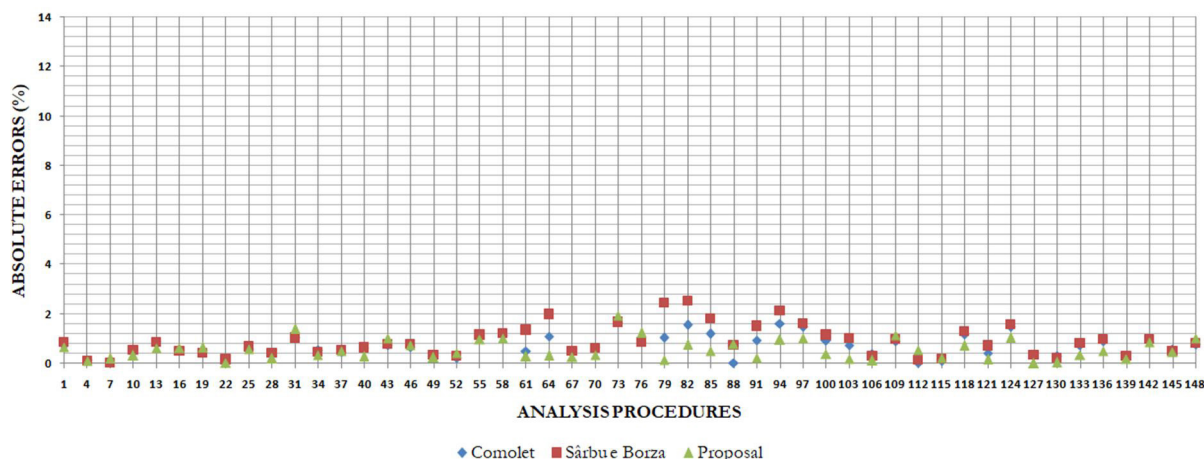|	Mean Absolute Errors (%)		
	Comolet	Sârbu	Proposal
Scenario 1	0.70	0.88	**0.55**
Scenario 2	1.62	1.95	**1.39**
Scenario 3	3.48	3.83	**2.86**

Figure 5. Absolute errors in the procedures of Scenario 1 (speed reduction in 10% of the nominal rotation).
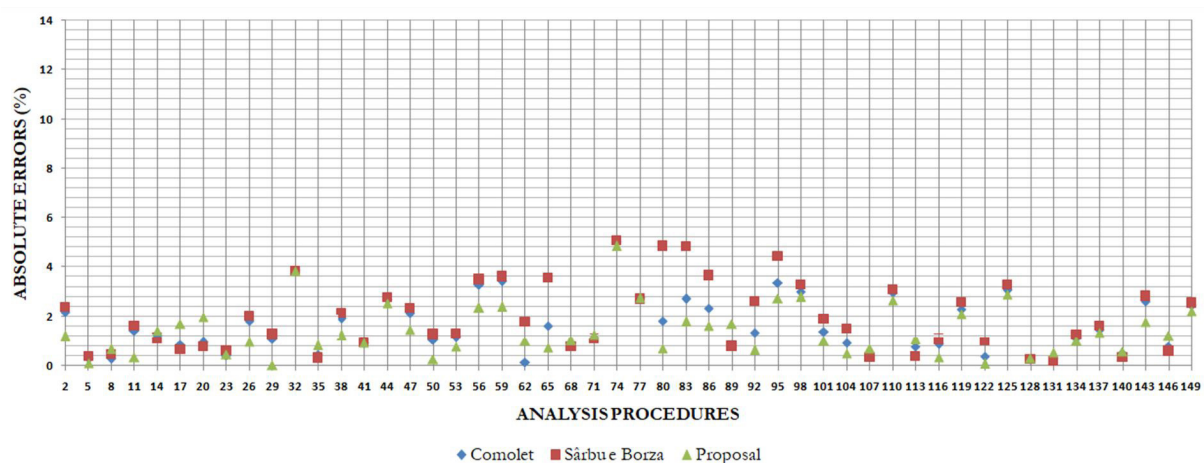

Figure 6. Absolute errors in the procedures of Scenario 2 (speed reduction in 20% of the nominal rotation).
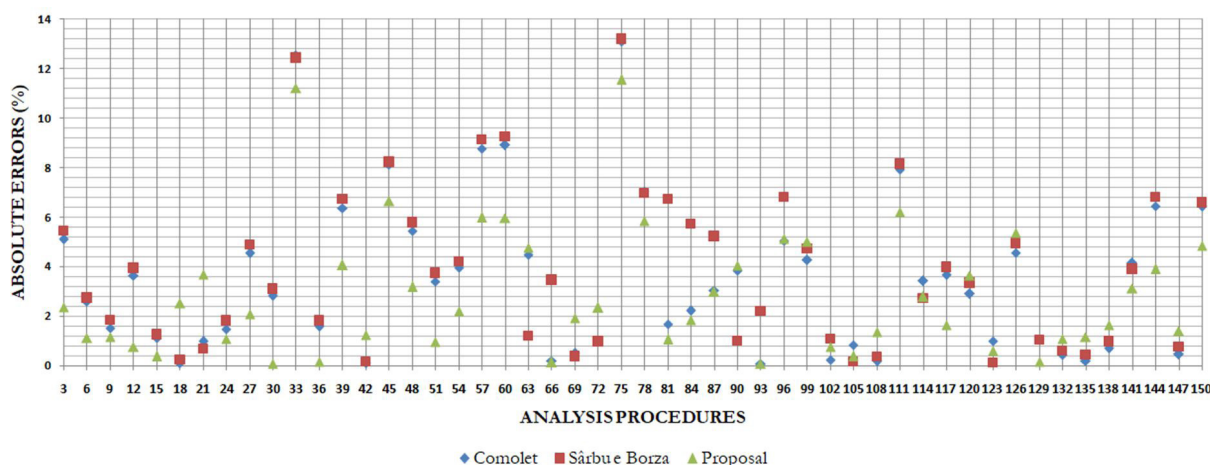

Figure 7. Absolute errors in the procedures of Scenario 3 (speed reduction in 30% of the nominal rotation).

As well as in Tables 2 and 4, it is also evidenced through the graphs that the error in the final efficiency estimates of the three formulas analyzed increases as far as the rotation speed gets distant from the nominal rotation. This is due to the fact that the final efficiency in the above-mentioned formulas depends on ratio R among the rotative speeds.

The superiority of the formula proposed, in terms of accuracy in the estimate of a pump's final efficiency, is also evidenced in the graphs from Figures 5 to 7. It is possible to notice that the characters related to it (▲) are most of the time below those associated to the other two formulas (♦ and ■).

It is worth highlighting that among all the analysis procedures executed, in 101 of them the equation here proposed has provided more accurate estimates than the equation proposed by Comolet and in 102 estimates which have been more precise than the ones from the equation proposed by Sârbu and Borza. It has also enabled to verify that in 95 of the 150 procedures the formula proposed in this work has provided efficiency values which have shown to be more accurate than the ones in both empirical equations which have also been tested.

CONCLUSIONS

The use of variable speed pumps in water supply networks implies the necessity of estimating the pump(s)' efficiency in its new operating point after its rotation is modified with the highest accuracy possible. Based on the fact that the electrical energy consumption in a pumping system is inversely proportional to its efficiency, this work has proposed a new alternative for the accomplishment of these estimates.

The formula developed has been assessed in occasions when the pump's speed has been reduced up to 70% of its nominal rotation. The results of the analyses accomplished have indicated a higher accuracy of the equation here proposed when compared to the formulas proposed by Comolet and also by Sârbu and Borza. This fact has been verified through the absolute sum and the mean absolute errors found in each analysis procedure and, also, through the number of procedures in which the equation's final efficiency estimate here proposed has been more accurate than those accomplished by the other two equations assessed.

In light of the exposed, the utilization of the equation developed in this research is, consequently, advantageous with relation to the other two equations, since the efficiency estimates accomplished by it from the analysis procedures have shown to be more accurate and, thus, it means that the calculation for power and total energy consumed will also be more accurate.

For future research, an analysis of the formula proposed in this paper for wider ranges of speed change is recommended. Having in mind that the calculus for the efficiency through the affinity laws becomes the more imprecise the higher the percentage of speed reduction is, it is suggested, for this analysis, the accomplishment of an experimental survey with commercial pumps of different nominal speed values in which is possible to compare the final efficiency values estimated by the formula proposed with the values measured experimentally.

Finally, it is worth mentioning that the tests' reliability degree is related to the accuracy in the obtention of the pumps' performance and efficiency curves. The correlations between H and Q and between η and Q can be measured through the correlation coefficient r^2, which is disposed in the graph with all the curves' equations. In this sense, the inaccuracy of the estimates raised in the tests is believed to be very low, which is due to the obtention of the curves' equations which are characteristic of the pumps, since the lower value of r^2 perceived in this stage, and showed in Table 2, has been equal to 0.957.

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Authors contributions

José Nilton de Abreu Costa: Conception of the work, execution of the experiments, demonstration of the formula and writing of the article.

Marco Aurélio Holanda de Castro: Conception of work, discussion of results, review of the article.

Luís Henrique Magalhães Costa: Development of the computational tool, discussion of results, review of the article.

João Marcelo Costa Barbosa: Definition of methodologies, discussion of results, review of the article.