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YURI LACERDA COSTA

MODELING BAYESIAN UPDATING WITH MANY NON-UPDATERS: THE CASE OF OWN SUBJECTIVE HOMICIDE VICTIMIZATION RISK

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Dissertação de Mestrado apresentada ao Programa de Pós-Graduação em Economia, da Faculdade de Economia da Universidade Federal do Ceará, como requisito parcial para obtenção do Título de Mestre em Economia. Área de concentração: Métodos Quantitativos em Economia.

Prof. Dr. José Raimundo de Araújo Carvalho Júnior

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A Deus.
Aos meus pais, Luiz Cláudio e Adáuria.
Ao meu irmão, Sued.

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RESUMO

Nosso principal objetivo neste estudo é investigar o papel da heterogeneidade na atualização, depois de um choque de informação, do risco subjetivo sobre vitimização de homicídio. Nesse sentido, os dados utilizados neste trabalho também atestam a superestimação do crime encontrada na literatura. A novidade é que os entrevistados receberam um choque de informação que consiste na taxa oficial de homicídios, mas a grande maioria deles mantém a mesma percepção inicial. Ao propor um modelo de *Update* Bayesiano permitindo que nenhuma atualização fosse realizada, dois modelos foram desenvolvidos: um Tobit modificado e um modelo Hurdle de dois níveis. Assim como em estudos anteriores, nossos resultados mostraram que poderíamos prosseguir com uma abordagem de *Update* Bayesiano. Ainda, quanto mais altas as respostas iniciais eram definidas, mais propensos os indivíduos estavam em proceder uma mudança de percepção. Além disso, fundamentalmente, pudemos racionalizar a decisão de não revisar as respostas seguindo um argumento de qualidade/credibilidade da informação percebida. Descobrimos que os participantes mais velhos e as mulheres são mais relutantes não apenas em alterar as respostas iniciais, mas também na escolha do nível da nova resposta, em caso de mudança. Outra conclusão feita foi que o nível educacional dos entrevistados era insignificante em nosso exercício. De fato, o nível educacional do entrevistador teve um papel fundamental em ambas decisões de mudança e magnitude de revisão. Finalmente, nossos resultados também levantaram fortes evidências sobre aspectos de homofilia. A ocorrência de uma correspondência em gênero entre entrevistadores e entrevistados teve o maior impacto sobre a decisão de mudar e na magnitude da atualização neste estudo.

Palavras-chave: Probabilidades Subjetivas. Expectativas de Vitimização. *Update* Bayesiano. Modelo Tobit. Modelo Hurdle de Dois Níveis.

ABSTRACT

Our main purpose in this study is to investigate the role of heterogeneity into the update of subjective homicide victimization risk after an informational shock. In this sense, the data used here also attests the crime overestimation found in the literature. The novelty is that our respondents faced an informational shock consisting in the official homicide rate, but the vast majority of them keeps the same initial perception. In proposing a Bayesian Update model allowing that no update takes place, two models were developed: a modified Tobit and a two-tiered Hurdle model. In accordance with previous papers, our results showed that we could proceed with a Bayesian Update approach. Also, the higher initial responses are set, more likely individuals are in proceeding a change in perceptions. Furthermore, fundamentally, we could rationalize a non-updating decision following a perceived informational quality/credibility argument. We found that older participants and females are more reluctant not only to change initial responses, but also to choose the level of the new response, in case of an update. In addition, respondents' level of education was insignificant in our exercise. In fact, interviewers' level of education had a key role in both the changing and updating magnitude decisions. Finally, our results also raised strong evidence on homophily aspects. The occurance of a matching in gender between interviewers and interviewees had a major impact on the decision to change and in the magnitude of the update in this study.

Keywords: Subjective Probabilities. Victimization Expectations. Bayesian Update. Tobit Model. Two-tiered Hurdle Model.

Contents

1 INTRODUCTION

The last four decades of economic research are characterized by major advances in applied microeconomics. Issues previously restricted to other social sciences were being included into economists' research agenda due to an increasing interest in interdisciplinary topics. This is the context in which the Economics of Crime emerged with the seminal paper of Becker (1968). In this paper, the author models the engagement in illegal activities as result of a rational decision following a cost/benefit analysis under a neoclassical perspective. Based on this idea, discussions on deterrence and punishment strategies were raised designing public policies for crime control.

Apart from criminals, delinquency also involves a victim. In this sense, our study recognizes that the experience of crime itself is not the only input being processed. Also, the fear of being victimized plays a significant role in welfare and it is not unusual the occurrence of psychological disorders simply caused by a surge in crimes, developing serious problems to individuals' health and economic capability. However, it is important to note that becoming a victim is a rare event specifically focused on certain socioeconomic groups. Thus, given the importance of fear and the weak probabilistic justification for such a severe concern, understanding the factors underlying crime expectations and how information affect those risk perceptions is a gap we want to fill in.

In this sense, the standard economic analysis would suggest the use of probability distributions in order to deal with the uncertainty and risk involved in this situation. Nevertheless, there is an open debate on which type of distribution should be used: objective or subjective. Works such as Gollier (2004) have been explaining risky decisions through the first option, while a growing line of research argues in favor of the last one. This defense is mainly motivated by the ideas of Behavioral Economics, a field initiated by Tversky & Kahneman (1974), among others, consisting in an intersection between Psychology and Economics.

Under this perspective, studies on consumers' behavior were being added into an already consolidated Theory of Choice, overcoming unjustified and unnecessary assumptions in many applications. This is the ground on which Manski (2004) presents a detailed defense in using subjective probabilities and measuring expectations in several situations such as schooling choices and perceptions of the returns to it. Hereupon, subjective perception of crime occurrence probabilities appears as an important tool for understanding victimization.

In summary, this dissertation, much in the spirit of the literature initiated by Viscusi (1979) in a labor market context, assumes that citizens are rational agents with imperfect information on the actual probability of becoming a crime victim. Nevertheless, they face information news about crime which is not necesserily credible on a daily basis in such a manner compatible with a learning process. Indeed, credibility is a key concept in our discussion and it is going to be explored in greater details from section (3) onwards.

In light of this context, we want to study subjective expectations in the context of victimization, investigating the role of informational shocks on posterior risk perceptions. More specifically, we aim to look into the factors working on the update of individuals' subjective victimization probabilities after they receive a bit of information regarding official crime rates. In order to perform this task, we seek to develop an estimable model able to accommodate both optimal decisions on changing or keeping the same initial perception with respect to subjective homicide victimization risk. This enables us to rationalize the non-use of new information, contributing to the literature when allowing the existence of non-updaters in a Bayesian Updating context. Furthermore, we search for observable characteristics in explaining a two-step decision: i) Changing or keeping initial perception; ii) Setting an update (if any). The empirical exercise is going to take place by means of heterogeneity in both sides of the informational interaction, i.e., sender and receiver, as well as in matching aspects between them.

As suggested by its name, the Bayesian approach — the one adopted in this dissertation — relies on the widely known Bayes's theorem whose major contribution is to propose a link between current and past beliefs based on further evidence. Hence, situations involving initial perceptions followed by informational shocks are ideally suited for such analysis. However, since this approach fundamentally deals with subjective probabilistic beliefs, two relevant concerns arise: i) Are individuals able to think in terms of probabilities? ii) Are those responses reliable? Both questions pose resistance in working with subjective expectations, and some works, such as Hogarth (1975), provide a criticism on this reasoning. Nevertheless, there is strong evidence supporting the Bayesian approach in situations where the respondents faced relevant life aspects.

This is the case according to Viscusi & O'Connor (1984). The paper aims to investigate if chemical industry workers learn about the risks they face on the job and how this understanding affects the reservation wages. The authors conducted a survey and collected information for 335 individuals of many different occupations to conclude that these workers update their probabilistic beliefs in a way which is compatible with a Bayesian procedure. This main result is also found in Viscusi (1985). The author argues that the Bayesian Update model is a useful optimization approach for analyzing economic behavior and, in some cases, even the existing bias can be predicted by applying the model itself.

An application of this technique in Health Economics is found in Smith & Johnson (1988) analyzing agents' ability in processing information about the health risks of being exposed to a harmful substance. The main message brought by this paper is that the intoxication information which was neither recent nor relevant to the individual had insignificant effects on risk perceptions. On the other hand, this information seen as more updated and relevant appeared to be positively correlated to the posterior probability response. The authors also list four basic characteristics that a behavioral model — in some sense, what we want to propose — must consider:

i) The risky event's importance for individuals interviewed;

ii) The role of prior beliefs regarding the risky event;

iii) The implications of new information on this process;

iv) The acquisition costs of this new information;

Throughout this guidance, Bernheim (1990) examines the evolution and impact of new information on the American Social Insurance benefits' expectations. The results showed that, at a first glance, the agents were not well informed about their benefits level, but when the pre-retirement period approaches, they tend to make better use of the information provided. Both Smith & Johnson (1988) and Bernheim (1990) support the use of available information in updating expectations. However, they also warn that not every message is going to be used; it all depends on the informational content relevance.

The data — and methodology, as it is going to be explained later on — used in this dissertation also presents strong evidence on this sense. It was collected through a survey conducted in the city of Fortaleza, Brazil, between October 01, 2011 and January 19, 2012.

Twenty interviewers applied a questionnaire to 4,030 citizens of Fortaleza dealing with subjective expectations about being a victim of homicide on the following twelve months from the interview day onwards, among other issues. Apart from its size and information for both interviewees and interviewers, perhaps the most distinguished feature of this sample in the context of Bayesian Update is its vast majority of individuals who kept the same expectation, regardless the fact that their perceptions were far from official rates. We will refer to this behavior as *Skepticism* and to these respondents as *Skeptical Agents*.

We found that around 95% of our participants decided not to move from initial perceptions, which is, by far, very different empirical evidence than what is found in the literature. Instead of questioning the rationality of individuals who decided not to update expectations in a learning context, we might be facing a skeptical agent in such a way that the informational content must be relevant enough to make a change in his perception. The results found in our study support this understanding. For example, in all models we proposed, the interviewers' level of education, a central variable concerning the informational quality, had positive coefficients into the changing decision. The same is implied by a matching variable in gender, whose positive coefficients tell us that informational noise reduction is favorable to an update.

The analysis presented in this dissertation is divided into six sections. Section (2) presents a brief discussion on subjective expectations and Bayesian Update. Then, it analyses the contributions of Viscusi & O'Connor (1984) seminal paper and its influence on Smith & Johnson (1988) with greater details. Finishing the section, a recent application of subjective expectations in Delavande (2008) is presented. Section (3) presents the data, emphasizing its advantages in comparison with previous papers and our major challenge: to deal with a very different sample in this context. Also, it brings an initial exploratory analysis, as well as a brief discussion on selection bias.

The econometric models are presented in section (4). Firstly, a detailed derivation of a linear model for updating decisions is exposed in subsection (4.1). An extension of Viscusi & O'Connor (1984) and Smith & Johnson (1988) with a multiple linear regression is found in subsection (4.2). Subsection (4.3) develops two models to overcome the theoretical difficulties raised by our data and section (5) shows all estimation results. Finally, section (6) concludes this dissertation, summarizing major messages and proposing further contributions.

2 LITERATURE REVIEW

2.1 Subjective Expectations and Bayesian Update

The origins of Statistical Theory were mainly motivated by games of chance. At that time, not only determining the chances of a particular game result, but also inferring, given past evidence, some aspect of the game itself were questions underlying the work of many important mathematicians in the eighteenth century. In this context, the greatest contribution of Bayes & Price (1763), which was later improved by Laplace (1785), consists in a way to link the probability of a hypothesis, given past evidence, with the probability of that evidence, given the hypothesis. Formally, the famous Bayes's theorem establishes that:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Despite an intense debate surrounding Frequentist and Bayesian approaches, it is out of our scope considerations of such nature in this study. However, since we are interested in updating subjective expectations, we recognize that the Bayesian structure emerges as a convenient interpretation for our purposes. Sivia (1996) presents a discussion that makes this connection more evident. The author argues that the probability we assign to the proposition *"it will rain this afternoon"*, for example, will depend on whether there are dark clouds or a clear blue sky in the morning; it will also be affected by whether or not we saw the weather forecast. In other words, individuals dealing with probabilities are assigning numbers to express beliefs on events occurrences, conditioning it to previous knowledge about the generating process.

Although the conditioning on information background is often omitted in calculations to reduce algebraic cluttering, we must never forget its existence. In this sense, we need a slight modification on notation to let explicit this fundamental Bayesian aspect: Probability is not an absolute entity; it is an individual belief intrinsically conditioned to an informational set.

$$P(A|B,I) = \frac{P(B|A,I)P(A|I)}{P(B|I)}$$

where I is the conditioning informational set. Assuming that A is the *Hypothesis* and B is the *Evidence*, and from the fact that $P(Evidence|I)$ may be omitted[1], we have:

$$P(Hypothesis|Evidence, I) \propto P(Evidence|Hypothesis, I)P(Hypothesis|I)$$

This expression represents the core of a *Bayesian Update* model and its terms are defined and interpreted as follows:

$P(Hypothesis|Evidence, I)$: ***Posterior***

The state of knowledge about the truth of a hypothesis given extra evidence about the event under consideration.

$P(Evidence|Hypothesis, I)$: ***Learning factor***

A maximum Likelihood function.

[1] See Sivia (1996) for further details.

$P(Hypothesis|I)$: ***Prior***

The state of knowledge (or ignorance) about the truth of a hypothesis before any further evidence about the event under consideration.

Given the connection between initial perceptions and further evidence, the Bayesian appealingness for situations where agents possess incomplete information and are exposed to new informational bits is straightforward. Nevertheless, a previous step for its usage is of great resistance in Economics and Econometrics since, traditionally, information not objectively observed has little credibility[2]. Advocates of such understanding are based in Samuelson (1938) defending that the only information needed and usable is that of behavior only[3]. Proceeding this way, it is possible to recover initial preferences and the risk of making wrong assumptions about the introspection process is avoided. Also, it prevents any inconsistency raised by possible differences between what is said and done by the decision maker.

In this sense, the standard approach treats uncertain events occurrences as following an objective and known probability distribution. Based on this idea, Neumann & Morgenstern (1947) shows the existence of a particular function form known as *Expected Utility Function*. This result is intensively conjured up when modeling choice under uncertainty and, whenever it is necessary to comment on the expectations formation process, the Rational Expectation assumption is taken, or, at least, bounded rationality. However, as pointed out by Pesaran (1987), Rational Expectations has not proven to be sufficiently convincing to analysis out of macroeconomic long-run even in its weaker version.

Manski (2004), accomplished an extensive survey referring to Manski (1993) in a returns return to the schooling context. The author argues that students face the same inferential problems that labor econometricians do. Therefore, since much debate is still in course among experts, it is implausible to assume that students share the same objective and correct distribution generating the returns.

The problem is that avoiding the assumption of unique objective probability existence, perfectly known and identically processed by all individuals, the standard analysis cannot take place. This is so because any specific choice result may arise by many different preferences and probabilities combinations. Hence, Manski (2004) presents a detailed defense in favor of using expectations self-reported in probabilistic terms, the so called *Subjective Probabilities*[4]. This data type could be used to validate or relax hypothesis used in the models.

In light of such perspective, several papers have successfully applied subjective probabilities in many situations where expectations are crucial in the analysis. For example, Manski have co-authored studies in income expectations (Dominitz & Manski (1997)), social security and retirement (Dominitz, Manski & Heinz (2002)), consumer confidence (Dominitz & Manski (2004)) among others. An application in survival expectations is presented in Hurd & McGarry (1995).

Once having raised the relevance of bringing subjective probabilities for discussion, criticisms about the reliability of this data type cannot be neglected. Thus, it is coherent that the first empirical studies had concentrated themselves primarily in situations involving critical aspects for individuals under study, such as serious risks of health or death.

[2]Much of this skepticism is explained by early data type usefulness criticism found in Machlup (1946) and Hart, Modigliani & Orcutt (1960).

[3]An example of this acquaintance in Econometric Theory is found in McFadden (1973).

[4]A first attempt to use subjective probabilities in Economics is due to Juster (1966).

This is the context in which W. Kip Viscusi makes important contributions applying the Bayesian Update structure to model updating risk perceptions.

2.2 Early Contributions

Viscusi (1979), who was interested in the relation between risks for health and physical integrity at work and labor market outcomes, found that workers' perceived risk has a positive correlation with the industry specific risk. Also, it shows that industries whose employees had higher risk perceptions presented higher wages as well, supporting the Compensating Differentials Theory. Therefore, understanding how risk perception evolves and how it is processed, shaping a new subjective probability distribution, emerges as a further step on the study of labor markets dynamics.

Interested in the relationship between perceived risks for health and physical integrity at work and labor market outcomes, Viscusi (1979) finds that workers' perceived risk has a positive correlation with the industry specific risk. Also, it shows that industries whose employees had higher risk perceptions presented higher wages as well, supporting the Compensating Differentials Theory. Therefore, understanding how risk perception evolves and how it is processed, shaping a new subjective probability distribution, emerges as a further step on the study of labor markets dynamics.

Viscusi & O'Connor (1984) investigates whether employees revise their risk perceptions and, if so, how the reservation wage revision under a new informational set would be. Due to the lack of available data on workers' risk perceptions evolution, the authors applied a questionnaire to 335 employees of three major American chemical industries. Workers were asked to mark their risk perception in a linear scale ranging from very safe to dangerous, with the US private sector accident and diseases rate presented as a reference point. Finally, answers were converted into probabilistic terms giving rise to the variable *RISK*.

The study indicates that workers' perceptions were consistent with the US private sector risk average, even though it is 50% above that of the chemical industry. Moreover, about one third of the sample believed having been exposed to risks above the national average. Although these results may seem contradictory, the authors argued that the national reference rate was based on accidents related to work safety, aspect in which the chemical industry had a favorable track record. Thus, these official statistics had underestimated the long-term impacts on workers health due to chemical substances exposure. This evidence suggests some degree of a learning process.

In addition, Viscusi & O'Connor (1984) presents some models seeking to explain income based on standard variables such as race, education and experience, as well as those related to risk. This exercise allowed to identify risk premiums on wages, where these differentials ranged from $ 258.4 to $ 788.6, depending on the risk variable used and the subgroup analyzed. Finally, the results were consistently significant, in accordance with previous papers.

Since they were interested in the labor market dynamics, the authors created three other variables: *QUITA*, *QUITB* and *TAKEA*. The first two are related to decisions of leaving the job without transaction costs of searching for a new position. They differ from each other only regarding whether individuals consider themselves *"inclined to make a genuine effort to find a new job with a new employer next year"* or *"strongly inclined (...)"*, respectively. *TAKEA* refers to the decision of repeating the choice in accepting the job, given the current level of information. Although the results indicated that 79%

of the sample would decide, without hesitation, to accept the same job, the probability of *TAKEA* assuming value 1 is negatively related to all risk variables. Similarly, *QUITA* had a positive relationship.

The first message of Viscusi & O'Connor (1984) is that the data is consistent with a model in which the choice of employment when facing risks is a learning process. Reservation wages grow as perceived risks increase, so that there is a risk premium keeping workers at their jobs until such compensation is not enough. From that point on, they decide to leave the job or, at least, not to accept the offer if this choice was put back.

The major contribution of this paper, however, is to investigate the informational shock effect on workers' perceptions, which is exactly what we want to do in a victimization context. Following this purpose, the authors presented each worker a label containing information regarding a chemical substance that would replace the old products used in everyday work activities. Then, workers were asked to repeat their responses under this new information. There were four label types: *CARB* (Sodium Bicarbonate), *LAC* (Lachrymator Chloroacetophenone), *ASB* (Asbestos) and *TNT*. As expected, the control group *CARB* had a significant reduction in reported risk levels, virtually eliminating the chemical risk. As for *LAC*, *ASB* and *TNT*, employees reported risk levels almost three times higher and the vast majority believed having been exposed to above-average risks.

Assuming that workers had a learning process which was consistent with a Bayesian approach, Viscusi & O'Connor (1984) models the initial risk perceptions, or the *priors*, as following a *Beta* distribution. This particular choice was made considering the great freedom allowed by this distribution, its ideal fit for independent Bernoulli experiments and the fact that a beta distribution is a conjugate prior[5]. Thus, the new labels were treated as additional observations from bernoulli trials about suffering or not a work accident. The distribution parameters are p and γ, the initial probability of an unfavorable outcome, i.e. *RISK*, and a term measuring this *prior* accuracy, respectively. After observing m unfavorable (accidents) and n favorable (non accidents) results, the paper shows that the *posterior*, p^*, is given by:

$$p^* = \frac{\gamma p + m}{\gamma + m + n} \tag{1}$$

The informational content of each label i depends on both the information accuracy, ξ_i, i.e. the equivalent $m + n$ observations implied by this information, and $s_i = \frac{m}{m+n}$, the proportion of unfavorable observations. Thus, a label i increases the worker perceived risk when s_i is greater than p_i and the correction magnitude is positively related to ξ_i. Replacing ξ_i and s_i in (1), the posterior probability, p_i^*, of a work accident after receiving a warning for the chemical i is given by:

$$p_i^* = \frac{\gamma p + \xi_i s_i}{\gamma + \xi_i} = \frac{\xi_i s_i}{\gamma + \xi_i} + \frac{\gamma p}{\gamma + \xi_i} \tag{2}$$

Therefore, from (2), an estimable linear form can be established as follows:

$$RISKl_i = \alpha_i + \beta_i RISK_i + u_i$$

where $RISKl_i$ refers to the risk perception for substance i after the informational shock and u_i is a random error term. This equation represents the Viscusi's linear estimable model of Bayesian Update.

[5]For further details, see Pratt, Raiffa & Schlaifer (1995).

The regression results show that the coefficient α_i has a positive value and β_i plays an important role, so that workers *posteriors* have been influenced by both *priors* and label contents, creating a reviewing process consistent with the Bayesian approach. This is the starting point for our initial estimable model presented in subsection (4.2). For now, if we want to follow this idea for victimization updating expectations, our β_i must also be significant.

Motivated by Viscusi & O'Connor (1984), Smith & Johnson (1988) investigates how a sample of households in Maine, United States, responded to information regarding risks associated with radon concentration in their homes and water supplies. Whereas radon problems in the area had been published widely since the 1970's, the authors seek to address the question more specifically. Besides distributing pamphlets with information about the health risks caused by this chemical substance, they informed each household under study the radon concentration in its residence. The empirical model proposed is:

$$R_p = W_0 R_0 + W_s R_s$$

where R_p is the *posterior* risk perception reported, R_0 is the *prior* and R_s is the inferred risk from reading the pamphlet and from radon concentration result. W_0 and W_s are simply weights assigned in the risk reported calculation.

The authors argued that many aspects may affect R_s; personal attributes may influence both the interpretation of information provided and the credibility deposited on it. Thus, for simplicity, all these influences were grouped into a single vector, $\mathbf{Z}$, where $f(\mathbf{Z}, \mathbf{A})$ is a function and $\mathbf{A}$ is the parameters vector describing the interaction with R_s. Similarly, the vector $\mathbf{X}$ adds variables determining the relative weights and the functions $g_0(\mathbf{X}, \mathbf{B}_0)$ and $g_s(\mathbf{X}, \mathbf{B}_s)$ act forming W_0 and W_s, where $\mathbf{B}_0$ and $\mathbf{B}_s$ are parameters vectors. Therefore,

$$R_p = g_0(\mathbf{X}, \mathbf{B}_0) R_0 + g_s(\mathbf{X}, \mathbf{B}_s) f(\mathbf{Z}, \mathbf{A})$$

This is the model used in Smith & Johnson (1988) and, for estimation purposes, a linear relationship between $\mathbf{Z}$ and $\mathbf{X}$ is assumed and its respective parameters given by:

$$R_p = a_0 + a_1 R_{0j} + \sum_{i=1}^{N} b_i X_{ij} R_{0j} + \sum_{k=1}^{M} C_k Z_{kj} + \sum_{i=1}^{N} \sum_{k=1}^{M} d_{ik} X_{ij} Z_{kj} + e_j$$

where N and M are the number of determinants taken for the construction of weights and to form R_s, respectively.

The sample of 230 observations is composed of two groups: the first one consists of households that had already been subjected to a lung cancer epidemiological study conducted at Maine Medical Center; the second one is a randomly selected control group. After receiving the informational pamphlet produced by the University of Maine and its home radon concentration result, the individuals were interviewed by phone and had their socioeconomic characteristics, risk perception and risk mitigating actions collected.

The analysis is based only on 117 observations, those households whose information provided is complete. Although questions may be raised about a possible selection bias, the authors argue that much of the sample loss is due to incomplete information on socioeconomic and behavioral variables not related to one's ability in providing a valid response. After resizing the responses regarding perceived risk to a 0–1 scale, R_0 and R_p were obtained. The other variables in the model are radon concentration, years living in the residence, time since receiving radon information, usage of different informational sources, decision to take risk mitigating actions, life expectancy, presence of cancer members at home and total number of years in which the individual smokes. The results bring up five central conclusions validating the use of available socioeconomic and environmental information in updating expectations:

i) Higher levels of exposure to the radon information provided have a positive impact on the *posterior* magnitude;

ii) Mitigating actions are negatively related to risk perceptions;

iii) Cancer victims are very likely to increase *posterior* perception and the weight associated with the radon concentration;

iv) Life expectancy and age exhibit statistically insignificant results;

v) Individuals perceive new information with a third of their inital expectations;

Somehow extending the discussion presented in Viscusi & O'Connor (1984) to include other information than *priors*, Smith & Johnson (1988) provides an important insight for our study. Since the extent on which information is perceived determines its use when updating expectations, we should be cautious with these aspects in our analysis. Also, it is important to note that, despite the empirical success of these articles, we cannot overlook studies observing some nonconformity between experimental or empirical results and its underlying theory. As pointed out by Tversky & Kahneman (1974) the literature generally recognizes several specific decision rules used unconsciously and not necessarily in an optimal way to solve complex problems.

As it is the case for homicide victimization, dealing with rare events is an even greater challenge for most individuals. As an illustration of this, Kunreuther et al. (1978) shows that risk-averse consumers decided not to purchase insurance against floods and earthquakes even though they had been largely subsidized to do so. However, despite the difficulty in modeling choice behaviors in this context, papers such as Lichtenstein et al. (1978) indicates a pattern: regarding risk perceptions, agents tend to overestimate low probabilities and underestimate those of high magnitude.

Anticipating part of the data discussion found in section (3), our empirical evidence is an example of this pattern. There is a huge gap between respondents' initial perceptions and official values. The sample *prior* mean is strikingly 943 times greater than the official rate for homicide. Viscusi (1985) shows that this result is exactly what one might expect from a Bayesian learning process as a final argument for using a Bayesian Update model in this context.

2.3 Recent Developments

More recent papers on subjective expectations focus primarily on the development of probabilistic elicitation mechanisms which are able to access beliefs in terms of probabilities and to take the extensive evidence about heuristics in its updating into account. Naturally, issues concerning expectation revisions add even more challenges than collecting only initial subjective probabilities. In this context, the researcher must be aware not only to the probabilistic consistency in both periods under study, but, crucially, on how this review take place.

In an experimental setting, El-Gamal & Grether (1995) finds evidence on the use of certain heuristics, such as *Conservatism* and *Representativeness*, in addition to the Bayes's rule in updating subjective probabilities. In general, the results showed that this rule is the most likely approach used by agents, but the researcher should be alert to the fact that it is not the only one. This message is also found in Dominitz & Manski (2011). Looking at expectation revisions on equity returns, this paper evidences an extensive heterogeneity on the updating process.

In light of these results, Delavande (2008) develops the *ERS* (*Equivalent Random Sample*), a metric for subjective expectation updates, trying to accommodate the conflicting results about the specific revision process used by individuals. This metric is developed under the idea that reviewing processes can be formalized as follows: Let P_i be the true probability of an individual i experience the occurrence of B, a binary event. Assume that i is unaware of the value of P_i, so that he has only a subjective expectation about it. Furthermore, let $f_{i,1} \in \Gamma$ be the distribution function of this initial subjective expectation, where Γ is the set of all distribution functions defined on the interval $[0, 1]$.

Similarly, let $f_{i,2} \in \Gamma$ be the distribution function of the final subjective expectation about P_i, after i receives the information $o_i \in I$, where I is the set of all information regarding B. Thus, the update is given by a function $U_i(,..) : \Gamma \times I \to \Gamma$ such that $U_i(f_{i,1}, o_i) = f_{i,2}$ and discussions about the specific revision rules are translated into particular assumptions on $U_i(.,.)$.

The metric proposed by Delavande (2008) is developed exactly in defining this function. The author establishes the *ERS*, $\pi_i(o_i)$, of individual i, with information o_i, as a random sample of binary events under P_i that would have generated, using Bayes's rule, the revision of $f_{i,1}$ to $f_{i,2} = U_i(f_{i,1}, o_i)$. Although we are not particularly interested on the specificities of subjective distributions, this initial formalization is a fundamental motivation for our modeling in subsection (4.2), as it going to be clear later on.

Thus, $U_i(.,.) : \Gamma \times I \to \Gamma$ is established as:

$$U_i(f_{i,1}, o_i) = \frac{Pr(\pi_i(o_i)|p) f_{i1}(p)}{\int_0^1 Pr(\pi_i(o_i)|z) f_{i1}(z) dz} \tag{3}$$

Delavande (2008) follows previous studies in considering that $f_{i,1}$ and $f_{i,2}$ are beta distributions. Doing so, given some restrictions on the distribution parameters, it finds a univocal determination of $\pi_i(o_i)$, overcoming the ambiguity in determining which updating rule was used. It all depends on the induced $\pi_i(o_i)$ sample in comparison to the one implied by the given information. If the samples are equal, then the individual is revising like a Bayesian. If $\pi_i(o_i)$ is bigger or smaller, the individual uses representativeness or conservatism, respectively.

As an empirical exercise, Delavande (2008) proposes an analysis of women's learning process on the effectiveness of a fictitious contraceptive method, M. Thus, let a sexually active woman, i, who has a probability $P_i \in [0, 1]$, unknown for i, to get pregnant in a rolling year if M is being used. The distribution of subjective probabilities on P_i at time t is given by $f_{i,t}$. Also, let $o_{i,t}$ be the information possessed by i at time t. Under these conditions, let $A_{i,t}$ be the probability of i getting pregnant using the method M, i.e.,

$$A_{i,t} = \int_0^1 z f_{i,t}(z) dz$$

A clear purpose in Delavande (2008) is to determine individuals's subjective distributions. This is crucial for obtaining $\pi_i(o_i)$. Thus, the elicitation strategy is to introduce a wording concept based on the *"strength of beliefs"*, determining three other points on the cumulative distribution. Finally, following Dominitz & Manski (1996a), the author employs a least-squares-criterion to fit respondents's answers to a specific individual beta distribution. This concludes the *prior* collection stage. For the updates, the same reasoning is employed, apart from the fact that participants are provided with new information. After obtaining the optimal parameters choice, the *ERS* determination is straightforward.

The sample used consists of 92 sexually active female undergraduate students. Initially, the respondents were presented to the new contraceptive method and had their expectations about its effectiveness collected. After this first stage, three more steps, each one consisting of fictitious pregnancies record, were followed. Then, conducting a series of tests on the responses probabilistic consistency, the author concluded that the sample exhibits a coherent learning process, respecting the main rules of probability.

Although equation (3) does not take participants' observable characteristics into account, this is an important part of the study and it is of great influence in our work. Delavande (2008) argues that the updating process might be affected by heterogeneity either by means of information interpretation or by the specific updating rule. Thus, an extension of (3) is given by:

$$U_i(f_{i,1}, o_i) = \frac{Pr(\pi_i(o_i, \mathbf{X}_i|p) f_{i1}(p)}{\int_0^1 Pr(\pi_i(o_i, \mathbf{X}_i)|z) f_{i1}(z) dz}$$

Assuming that $B(p; a, b)$ is the beta cumulative distribution, the *ERS*, $\pi_i(o_i, \mathbf{X}_i)$ between stage t and $t+1$ for an individual with characteristics $\mathbf{X}_i$ is given by: $[a_{t+1}(\mathbf{X}_i) - a_t(\mathbf{X}_i), b_{t+1}(\mathbf{X}_i) - b_t(\mathbf{X}_i)]$, where $a_t(\mathbf{X}_i)$ and $b_t(\mathbf{X}_i)$ are the parameters of the time t fitted beta distribution. Given the positiveness required, the author proposes:

$$\begin{aligned}
a_t(\mathbf{X}_i, \alpha_t) &= \exp(\alpha_t \mathbf{X}_i) \\
b_t(\mathbf{X}_i, \beta_t) &= \exp(\beta_t \mathbf{X}_i) \\
a_{t+1}(\mathbf{X}_i, \alpha_t, \sigma_t) &= \exp(\alpha_t \mathbf{X}_i) + \exp(\sigma_{t+1} \mathbf{X}_i) \\
b_{t+1}(\mathbf{X}_i, \alpha_t, \delta_t) &= \exp(\beta_t \mathbf{X}_i) + \exp(\delta_{t+1} \mathbf{X}_i)
\end{aligned}$$

Obtaining $\hat{\alpha_t}$ and $\hat{\beta_t}$ is by solving:

$$\min \left[\sum_{i=1}^{92} L_{it}(a_t(\mathbf{X}_i, \alpha_t), b_t(\mathbf{X}_i, \beta_t)) \right]$$

where, $L_{it}(a_t(\mathbf{X}_i, \alpha_t), b_t(\mathbf{X}_i, \beta_t))$ is the least-squares fitted beta distribution.

Similarly, for $\hat{\sigma_t}$ and $\hat{\delta_t}$,

$$\min \left[\sum_{i=1}^{92} L_{it}(a_t(\mathbf{X}_i, \hat{\alpha_t}, \sigma_{t+1}), b_t(\mathbf{X}_i, \hat{\beta_t}, \delta_{t+1})) \right]$$

The results show that education and familiarity with contraceptive methods induce women to update less, giving more weight to prior beliefs. On the other hand, older women and those who already use a hormonal method updated their beliefs more frequently. Those are the conclusions when looking at all stages. Nevertheless, an interesting initial result is a high number — in fact, the majority — of women who do not update their beliefs at a first moment, i.e. they do not change perceptions until more evidence is provided in the following stages.

Delavande (2008) does not pay too much attention to this. In fact, although the author discusses some aspects on the updating *decision*, this is not of particular interest. The paper is more concerned to propose a metric explaining the updating *mechanism* and, even when there is no change in beliefs, the *ERS* captures the mechanics of such decision. Also, since two more changing opportunities follow successfully, there was no reason to discuss this evidence in detail. But what if this first changing data is the only one available? What if explaining the updating decisions is the goal?

The *ERS* was designed to access posterior subjective probability distributions. Nevertheless, the idea behind it can also be used to discuss informational perceptions and/or credibility. Why the information received should always be used? We believe the decision and magnitude of an update is critically dependent on the information quality and a no-changing decision perhaps is the best evidence to explore this idea.

3 DATA

Before starting the sample description, we want to emphasize some key aspects that make us believe in its uniqueness. Firstly, we are dealing with crime risk perception data, a type of empirical evidence that is hard to find in the literature of Risk and Information, despite its relevance for social welfare purposes. Furthermore, our sample relates to a developing country, and it is well known that obtaining microdata in such societies challenges researchers of all fields. Focusing on crime issues, according to a recent study conducted by the United Nations[6], Brazil, the country on which our data relies, shows one of the highest homicide rates in the world, representing a significant concern for public policies. If one wants to study crime perceptions, our data offers rare information of individuals living in a pertinent environment.

Secondly, as to subjective perceptions, the amount of observations is far more than what is often presented in some important papers in the field. For example, Viscusi & O'Connor (1984) presents no more than 400 respondents in a seminal paper. Our sample is more than 10 times greater than this, presenting responses of individuals from all of the 116 districts of Fortaleza with an ample range of age, education and income, to name some of the available information.

In addition, unlike Delavande (2008), who conducted herself the survey, we have twenty different interviewers. This is crucial to avoid some potential, and very often unconscious, problems emerging from this data type being collected by the same person who is going to analyze it. Moreover, if we want to model information use and updating decisions, this issue may introduce unnoticeable selection bias due to the fact that informational source does not change. More importantly, the variation on the sender's side, raising differences in gender, age and educational levels, enables to investigate aspects of matching with no concerns of self selection, since both participants were not able to choose each other[7].

After a cleaning and imputation procedures in our initial data set, we have a sample of 2885 complete observations. Table (1) presents all variables we are going to use in this study. Overall, the big picture we are going to discuss in detail is depicted by a middle-age, non-white and female sample, with low income and educational levels. In other words, it is a relevant representation of Fortaleza's residents.

Out of 4,030 observations, 2190 are women and 1840 are men with an average age slightly more than 39 years old. Due to this wide amplitude in age — the youngest citizen is 16 and the oldest is 94 years old —, it is informative to say that the median is 37 years old. With respect to marital status, we have 1928 single individuals, while the portion with a partner sums 2102. The respondents were also able to identify themselves as White, Black, Mestizo, Asian or Indigenous. Grounded on the standard approach, we classified our sample into White and Non-White, finding only 31% as Caucasian. In addition, respondents could choose between 8 levels of education and income. Following the Brazilian high income concentration and low educational level, the majority of our sample has 1 to 2 minimum wage as household income and an incomplete elementary school degree.

Getting into our major focus, which concerns the probabilities reported, after a preliminary training in basic probability concepts[8], participants faced an initial question posed as:

[6]United Nations (2013).

[7]The interview was held in a randomized household and interviewers were also randomly allocated.

[8]This training consisted in stating the range limits and examples of a coin toss and balls in a box.

Table 1: Variable Definitions

Variable	Description	Range
Prior	Initial subjective probability about becoming a victim of homicide on the following 12 months from the interview day onwards.	[0,100]
Change	A dummy for respondent decision to change initial perception.	0: No; 1: Yes.
*Post**	Final subjective probability about becoming a victim of homicide on the following 12 months from the interview day onwards.	[0,100]
Gender	A dummy for respondent's Gender.	0: Male; 1: Female;
Age	Respondent's age in years.	[16,94]
Police	A level variable assessing the police work in terms of crime control around the area of respondent residence.	1: Excellent; 2: Good; 3: Fare; 4: Poor; 5: Terrible.
Educ_Int	A dummy for interviewer's educational level.	0: High School; 1: More than High School.
Matching_Gender†	A dummy for the existence of a gender matching between respondent and interviewer.	0: No; 1: Yes.

* Note that, depending on *Change*, $Post = Prior$ or $Post \neq Prior$.

† Note that it does not matter what specific gender matching is observerd.

Source: Elaborated by the author.

"Regarding numerical values, what is the chance (or probability) of you being a victim of the following crimes in Fortaleza in the next 12 months: a) Homicide; b) Robbery".

Despite the controversy surrounding human ability to think probabilistically, and in accordance with Ferrell & McGoey (1980) and Koriat, Lichtenstein & Fischhoff (1980), only a bit more than 1/4 of our sample is not able to give a valid response to this question, which is identified as either "*Didn't Know*" or "*Didn't Answer*" and eliminated from our database. We consider this result satisfactory and, due to a large data set, we have a great number of observations to work with: 2885. We will refer to this first response as *Prior*.

The interviewer, after receiving this initial probabilistic perception, gives an informational shock as follows:

"According to the Ministry of Justice, in 2009 the probability of a person in Fortaleza: a) being victim of homicide was 0,037% (37 homicides for each 100.000 inhabitants) and b) being robbed was 0,96% (960 thefts for each 100.000 inhabitants)".

Then interviewees are asked if they want to change their initial responses and, if so, to what value. We call this decision to change as *Change* and the posterior perception as *Post*.

Table (2) summarizes the first sample statistics. We have already anticipated comments on *Gender* and *Age*. However, our empirical exercise is also going to include environmental and matching issues, as well as interviewers's characteristics. We argue that our interaction outcomes rely not only on the respondent's side, but also on the sender's side and on common features between both of them. Since we are trying to model perceived information, we should consider message quality aspects and a first natural choice is the interviewers' level of education. All of our 20 questioners, at least, completed high school studies and 80% of them are undergraduate students or graduates.

Table 2: Descriptive Analysis – Homicide

	Mean	Std.dev	Min	Max	Missing
Prior	34.91	26.49	1.00	98.00	0
Change	0.04	0.21	0	1	0
Post	33.29	26.63	0.10	98.00	0
Gender	0.53	0.50	0	1	0
Age	38.55	16.23	16	94	56
Police	2.95	0.96	1	5	22
Educ_Int	0.83	0.37	0	1	0
Matching_Gender	0.77	0.42	0	1	0

Source: Elaborated by the author.

Equally, sources of information noise are concerns to take into account. Note that differences among interviewers and interviewees possibly raise obstacles on information, and it works against an update. In this sense, an important concept to this analysis is known as *Homophily*. McPherson, Smith-Lovin & Cook (2001) defines it as the contact between similar people occurring at a higher rate than among dissimilar people. It is important to keep in mind that, at first, homophily can be related to race, gender, age, religion, social status or any other similar characteristic relating individuals. For example, whenever a group exhibits one of these heterogeneities at a higher rate than it would be found randomly, there is homophily.

The impact of these differences on spontaneous groups' formation and networks ties is widely documented since the early 1920's. For example, Bott (1928) noted that school children formed friendships and play groups at higher rates if they were similar on demographic characteristics. The implications of homophily to information diffusion is a research agenda of recent papers such as Jackson (2009) and Golub & Jackson (2012). Both studies attest that, if a society is divided into several groups with a strong homophily within each one of them, i.e. contundent differences among groups, the convergence speed in which information spreads is decreased.

Using this result in our problem, it is possible that the interaction is affected by a particular heterogeneity between interviewers and interviewees. We conjecture that, in Latin cultures, gender plays a significant role on information credibility/use. Hence, *Matching_Gender* was created to control this issue. In addition, given our interest in victimization risk perception, the way respondents evaluate police forces is a very important piece of information to control for. In a range of five levels, individuals rated the police work around its middle point, which stands for Fare.

Finally, our subjective probabilities, i.e. *Prior* and *Post*, take almost the full range $[0, 100]$ with a huge variance. This initial result piles up to the empirical evidence presented in Manski (2004), refuting usual concerns about subjective probabilities being reported in values around 50%. Also, as in Delavande, Giné & McKenzie (2011), we refute the fears that poor, illiterate individuals in developing countries do not understand probability concepts. Equally, the significant difference between respondents's initial perceptions and the official rate could be another source of distrust in such data type. Nevertheless, it finds reverberation in previous papers such as Dominitz & Manski (1996b) and Jr, Grasmick et al. (1999) also attesting the existence of a crime overestimation.

The fact is: although a high discrepancy between subjective victimization probabilities and official rates is not new, our participants were informed about the "true" probability and still less than 5% of them changed initial perceptions. Actually, following Hoffrage et al. (2000), we presented the same information in numbers (*37 in 100.000*), which brings better results in the participants's use of probabilities. Therefore, given that initial perceptions are so overrated, what could explain almost no change in responses?

Perhaps looking at our updaters only provides some insight on this issue. It is important to keep in mind that we have two different steps after individuals receive an informational shock: i) Whether or not to change initial responses; ii) Given that a change is going to happen, to what value it is going to be set. In a sample of 2885 responders, 2758 of them say no to the first question and 127 behave accordingly to previous studies on Bayesian updating processes. Since we have a reasonable updating subsample size here, we can restrict our analysis to this data part and look for any noticeable difference between updaters and the entire sample. Table (3) presents the results.

Table 3: Descriptive Analysis – Updating Subsample

	Mean	Std.dev	Min	Max	Missing
Prior	39.55	28.43	1.00	90.00	0
Change	1	0	1	1	0
Post	2.64	2.70	0.10	15.00	0
Sex	0.54	0.50	0	1	0
Age	33.95	14.63	16	78	6
Police	2.99	0.86	1	5	0
Educ_Int	0.91	0.29	0	1	0
Matching_Sex	0.93	0.26	0	1	0

Total of 127 observations.

Source: Elaborated by the author.

Even though we are dealing with a critical question, in which anchoring in expectations is known to be quite frequent, and the amount of information provided was not ideal, we find that individuals who decide to reevaluate their responses do it in a scathing way. In this case, the subsample set *Post* almost 15 times less than *Prior*. This evidence suggests that our conjecture about the impact of senders' educational level and gender matching into message quality might be on the right path. When we look back to table (2) and compare it with table (3), we find that *Educ_Int* and *Matching_Gender* means increased considerably from 83% and 77% to 91% and 93%, respectively, in our updating subsample. Furthermore, *Prior* and *Age* are other variables that this crude analysis indicates to be aware of. The results show that our updaters are younger individuals with higher initial perceptions.

In summary, this very simple exercise induces us to believe that a selection bias might be working behind the scenes. Note that the *Post* reported is conditioned to a previous decision and, when we select our sample only for individuals who decide to change initial responses, this subgroup has some different characteristics from the entire sample. Those characteristics might be just what we are looking for.

Remember that our main purpose in this study is to investigate the role of heterogeneity into the update of subjective homicide victimization risk after an informational shock. Guiding us to this task, Viscusi & O'Connor (1984) shows the existence of an estimable simple linear regression, using *priors* to explain *posteriors*. Also, Delavande (2008) provides us some insights on how to propose a Bayesian Update model allowing the possibility of skepticism, i.e. setting $Prior = Post$.

In short, the data used here also attests the existence of a crime overestimation found in previous works. The novelty is that our respondents faced an informational shock consisting in the official homicide rate, but 95% of them keep the same initial perception. So far, looking at the updating subsample only, we found that these updating interactions were made by more educated interviewers, sharing the same gender with our interviewees. Thus, those two most important results enable us to follow our main line of explanation: informational quality.

In the next section, we start discussing our models. Initially, as a motivating exercise of what will follow, subsection (4.1) presents a classical Bayesian Update model. Subsection (4.2) is the first step of our contribution to extending Viscusi & O'Connor (1984) and Smith & Johnson (1988) proposing a multiple linear regression motivated by victimization expectations. We believe subsection (4.3) is our main achievement. It proposes two alternative models rationalizing the skepticism found in our data and it provides an extension of current models in allowing the existence of skeptical Bayesian updaters. The estimation results are presented separately for each model in section (5). Lastly, section (6) concludes this dissertation.

4 ECONOMETRIC MODELS FOR BAYESIAN UPDATING

4.1 Classical Bayesian Update

As mentioned in subsection (2.1), the starting point of the Bayesian Theory is given by equation:

$$P(Hypothesis|Evidence) \propto P(Evidence|Hypothesis)P(Hypothesis)$$

As a first motivating empirical exercise, consider the case of a random experiment composed by three coin tosses *iid*. Success, $y = 1$, consists of getting a head, with no loss of generality and let θ be the unknown success probability. We want to use the tosses result, say $R = (1, 0, 1)$, to infer about the true value of θ.

The parameter θ is treated as an unknown parameter about which the individual has a first approximation. Then, he will use evidence R to make an update over the true value. Formally,

$$p(\theta|R) \propto p(R|\theta)p(\theta) \tag{4}$$

where p represents the probability distribution function at a Bayesian context.

Given the structure proposed,

$$p(R|\theta) = \theta^2(1-\theta) \tag{5}$$

As for the heart of a Bayesian approach, we must admit an initial probability distribution for θ. It represents the observer uncertainty about the population value. Following Pratt, Raiffa & Schlaifer (1995) in the choice of a *Beta* distribution for *Bernoulli* processes, we assume that $\theta \sim Beta(\alpha, \beta)$, i.e.,

$$p(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1} \tag{6}$$

Substituting (5) and (6) into (4),

$$p(\theta|R) \propto \theta^2(1-\theta)\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}$$

i.e.,

$$p(\theta|R) \propto \theta^{\alpha+1}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(1-\theta)^{\beta} \tag{7}$$

Then, $\theta|R \sim Beta(\alpha+2, \beta+1)$. For the predicting exercise of $y = 1$ given R, we need $p(y = 1|R)$. So,

$$\begin{aligned} p(x=1|R) &= \int p(y=1,\theta|R)d\theta \\ &= \int p(x=1|\theta,R)p(\theta|R)d\theta \\ &= \int \theta p(\theta|R)d\theta \\ &= E(\theta|R) \end{aligned}$$

Then, the probability of getting a head in the next toss, given the past evidence, is:

$$P(y=1|R) = p(y=1|R) = E(\theta|R)$$

Therefore,

$$P(y=1|R) = \frac{\alpha+2}{(\alpha+2)+(\beta+1)}$$

Generalizing the number of tosses,

$$\theta|(R_1, R_2, ..., R_N) \sim Beta(\alpha + \sum_{i=1}^{N} R_i, \beta + N - \sum_{i=1}^{N} R_i)$$

$$P(y=1|(R_1, R_2, ..., R_N)) = \frac{\alpha + \sum_{i=1}^{N} R_i}{(\alpha + \sum_{i=1}^{N} R_i) + (\beta + N - \sum_{i=1}^{N} R_i)}$$

Using this same reasoning, we will start our victimization expectation modeling. The *Prior* collection is very similar to what we did here, but we introduce heterogeneity in the discussion. Then, we look a bit closer to the role of observable and unobservable characteristics into the updating process, providing the first structural model. Finally, in assuming a linear form for the updating function, we propose our first estimable multiple linear model.

4.2 Multiple Linear Regression and Victimization Expectation Update

Consider a victimization event, where $y = 1$ if the individual becomes a crime victim and $y = 0$ on the contrary. Let $Y \sim Bern(\pi)$, *iid*, and assume that the respondent has a probability distribution $Beta(\alpha, \beta)$ about the true, but unknown, probability π of becoming a homicide victim.

In this fashion, we propose the following interaction structure: the initial respondent's perception, $Prior_i \equiv \pi_i^0$, can be modeled as dependent on his observable characteristics, for instance, age, gender, income, education, etc. Formally, we can define a function $f : \mathbb{R}^q \to [0,1]$ such that $\pi_i^0 = f(\tilde{\mathbf{X}}_i^r)$, where $\tilde{\mathbf{X}}_i^r$ is the q dimensional heterogeneity vector of individual i. When asked about his initial perception, the individual assess $P(y=1|\tilde{\mathbf{X}}_i^r) = E(\pi|\tilde{\mathbf{X}}_i^r) = \pi_i^0$ and establishes as response $\pi_i^0 = \frac{\alpha_i}{\alpha_i+\beta_i}$.

After collecting π_i^0, the interviewer sends an informational bit consisting in the population value, π. However, from the respondent's perspective, this information may lack credibility and its content may not be fully taken into account. In fact, let π_i^* be the proper value depicted by this information and define $g : \mathbb{R}^k \to [0,1]$, with $\pi_i^* = g(\mathbf{X}_i^r, \mathbf{X}_j^s, \mathbf{X}_{i,j}^m, \pi_i^0)$. In this function, $\mathbf{X}_i^r$ and $\mathbf{X}_j^s$ represent the vectors of observable heterogeneity for receivers[9] and senders, respectively, and $\mathbf{X}_{i,j}^m$ is a vector of matching aspects.

Abstractly, the information was translated into a sequence of victimization observations, generating an informational content $\mathcal{I} = (0, 1, \ldots, 0, 1)$ with n_1 values equal 1 and n_0 equal 0, where $N = n_0 + n_1$. Finally, the respondent, whose initial perception was π_i^0, uses the perceived information, π_i^*, to generate $\mathcal{I}_i$, creating a set of new evidence which is the foundation to update his response towards a posterior perception. This new response, $Post \equiv \pi_i^p$, is given by $P_i(y=1|\mathcal{I}_i) = E(\pi|\mathcal{I}_i) = \pi_i^p$.

[9]The vector determining π_i^0 is not necessarily the same for π_i^*.

In this structure,

$$\begin{aligned} p_i(\pi|\mathcal{I}_i) &\propto p_i(\mathcal{I}_i|\pi)p_i(\pi) \\ &\propto \pi^{n_1}(1-\pi)^{n_0}\frac{\Gamma(\alpha_i+\beta_i)}{\Gamma(\alpha_i)\Gamma(\beta_i)}\pi^{\alpha_i-1}(1-\pi)^{\beta_i-1} \end{aligned}$$

Then,

$$\pi|\mathcal{I}_i \sim Beta(\alpha_i+n_1, \beta_i+n_0)$$

And,

$$\pi_i^p = E(\pi|\mathcal{I}_i) = \frac{\alpha_i+n_1}{\alpha_i+\beta_i+n_1+n_0}$$

For estimation purposes, π_i^p remains undefined, since it is not possible to identify none of the given parameters. One way to dodge this problem is to realize that $\pi_i^* = \frac{n_1}{N}$ and use the fact that $\pi_i^0 = \frac{\alpha_i}{\alpha_i+\beta i}$ to establish:

$$\pi_i^p = \frac{N(\frac{n_1}{N}) + (\alpha i+\beta_i)\frac{\alpha_i}{(\alpha_i+\beta_i)}}{\alpha_i+\beta_i+N}$$

Therefore, we are able to write our structural model as:

$$\pi_i^p = \frac{N}{\alpha_i+\beta_i+N}\pi^* + \frac{\alpha_i+\beta_i}{\alpha_i+\beta_i+N}\pi_i^0 \tag{8}$$

Now, if we assume that $g: \mathbb{R}^k \to [0,1]$ is given by a linear form, it allows us to extend Viscusi & O'Connor (1984) and Smith & Johnson (1988) at the same time by taking not only respondent's heterogeneity into account, but also both interviewer and matching information through a coherent conditioning multiple linear regression as follows::

$$\pi_i^p = \lambda + \gamma\pi_i^0 + \mathbf{X}_i\delta + \epsilon \tag{9}$$

where $\mathbf{X}_i$ is an extended vector composed by $\mathbf{X}_i^r, \mathbf{X}_i^s, \mathbf{X}_i^m$.

4.3 Bayesian Updating with Skepticism

As explored in section (3), we have a very different sample in the context of Bayesian Update. Our data presents an excess of individuals – our skeptical agents – who do not update and keep their posterior responses with the same value as their initial perceptions, i.e. $Prior_i = Post_i$. One might point this issue as an important downside of our study. In fact, previous works present data collection procedures specifically designed to assess subjective probabilities and its revision processes[10]. On the other hand, our dataset comes from a broad household survey.

Clearly, individuals faced a critical question – homicide expectation – about which initial perceptions were too far from the truth. In addition, the information provided lasted less than a couple of minutes. However, even under such circumstances, if we proceed the estimation of equation (9) considering only those 127 individuals who changed initial responses, we will work on a sample almost 40% greater than what is presented in Delavande (2008), for example. Indeed, this exercise is part of our estimation procedure presented in subsection (4.3.2).

[10]Delavande (2008) and Delavande, Giné & McKenzie (2011) are good examples of this.

By now, we want to extend the discussion on informational content raised by Viscusi & O'Connor (1984) to move from an extreme, where almost every participant revises their initial perception, to the other, where the contrary occurs. Is that reasonable to keep considering an economy composed by agents who always believe in the information received for every risky decision to be made? Putting in other words, is all information provided about uncertain outcomes informative enough to make almost every individual change initial perceptions? Our dataset sends a clear message: for victimization expectations this is not the case and restricting attention only to those updaters would lead us, at least, to waste a huge amount of information. Also, these statistics reported might be misleading, due to potential selection bias.

In this study, it is important not to neglect individuals who did not respond to new information, since they carry messages about the updating process. Still, how to accommodate rational individuals in the same model facing the same information but behaving so differently? At this point, it is clear for us that our venture and major contribution is to propose a framework compatible to a Bayesian Update allowing the possibility that no update takes place. The main question is: could we build an econometric model which was able to estimate the impact of $Prior_i$ on $Post_i$ and, at the same time, consider the fact that $Prior_i$ is a threshold point for the existence of an update? In this sense, we seek to develop an econometric model which is able to address the following issues:

i) Agents are Bayesian when updating their victimization expectations;

ii) Observable and non-observable heterogeneity must refer not only to the interviewee but also to the interviewer, since matching aspects may influence credibility and information usage;

iii) The presence of non updating individuals must receive an adequate treatment and be rationalized under a Bayesian context.

With that in mind, rewrite (8) as follows:

$$\pi_i^p = \Big(1 + \frac{\alpha_i + \beta_i}{N}\Big)^{-1}\pi^* + \Big(1 + \frac{N}{\alpha_i + \beta i}\Big)^{-1}\pi_i^0 \tag{10}$$

Note that, from π_i^0, we may interpret $\alpha_i + \beta_i$ as the quantity of samples initially used to form $Prior_i$. Following this reasoning, $\eta_i = \frac{N}{\alpha_i+\beta_i} \in [0, \infty)$ may be interpreted as a measure of the informational quality. In this way, (10) can be rewritten as:

$$\pi_i^p = \Big(1 + \frac{1}{\eta_i}\Big)^{-1}\pi^* + \Big(1 + \eta_i\Big)^{-1}\pi_i^0 \tag{11}$$

This structural form will be crucial to skepticism rationalization. We need to explain individuals setting $\pi_i^p = \pi_i^0$, and it is reasonable to assume that the decision to revise perceptions, i.e to make $\pi_i^p \neq \pi_i^0$, is directly related to the credibility and quality of the information obtained. As (11) makes clear, when η_i approaches zero, or the more irrelevant the information is seen by the decision maker, π_i^p approaches π_i^0, which, in the limit, translates into the complete non-existence of an update. In the other extreme, when η_i approaches the infinity, π_i^p takes exactly the value π_i^*.

In summary, individuals setting $Post_i = Prior_i$ tell us that informational content was irrelevant and there was no reason to change initial perceptions. Ignoring these responses introduces a selection bias since establishing no changing in perceptions is a rational decision based on an intrinsic optimization process, which might be dependent on observable characteristics. Therefore, technically, there are observable and unobservable characteristics influencing a previous decision, subsetting our sample non-randomly.

In this situation, as pointed out by Wooldridge (2010), it is hard to believe that the now restricted error term has a zero conditional mean and, even if the structural model is linear, OLS procedure leads to inconsistent parameter estimates. The appropriate approach to deal with these cases is to think of an unrestricted and unobservable *latent* variable underlining the true observations through a specific structure. That is exactly what we will develop next.

4.3.1 Model 1: A Generalized Tobit for Bayesian Updating

Initially, let Y be a random variable of interest and Y^* its latent counterpart. then,

$$\begin{aligned} Y_i^* &= \mathbf{X}_i\delta + u_i, \qquad\qquad u_i|\mathbf{X}_i \sim Normal(0,\sigma^2) \\ Y_i &= max(0, Y_i^*) \end{aligned}$$

where $\mathbf{X}_i$ is a vector of conditioning variables.

This model, originally motivated by Tobin (1958), is known as the *Standard Censored Tobit Model*. Adapting it for our purposes, we assume the existence of a latent variable $Post_i^*$ that will govern the censoring mechanism. What is $Post_i^*$? We conjecture that $Post_i^*$ captures the respondents' latent *posterior* probability and, as such, according to equation (9), we propose it is a function of the *prior* and of an observable variables vector regarding both interviewees and interviewers. Hence, consider the following Tobit:

$$Post_i^* = \gamma Prior_i + \mathbf{X}_i\delta + u_i, \qquad u_i|Prior_i, \mathbf{X}_i \sim Normal(0,\sigma_u^2) \tag{12a}$$
$$Post_i = \min(Prior_i, Post_i^*) \tag{12b}$$

But why the minimum? This is so because, in our sample, all initial perceptions are greater than the official rate. Thus, for the sake of rationality, we must have $Post_i \leq Prior_i \; \forall i$. Now, note that:

$$Post_i = \min(Prior_i, Post_i^*) = -\max(-Prior_i, -Post_i^*)$$

However,

$$\max(-Prior_i, -Post_i^*) = \max(0, Prior_i - Post_i^*) - Prior_i$$

Then,

$$Post_i = Prior_i - \max(0, Prior_i - Post_i^*)$$

This means that our Tobit is equivalent to the following "new" Tobit:

$$Prior_i - Post_i^* = (1-\gamma)Prior_i - \mathbf{X}_i\delta + v_i \tag{13a}$$
$$Prior_i - Post_i = \max(0, Prior_i - Post_i^*) \tag{13b}$$

where, as usual, we assume that $v_i|Prior_i, \mathbf{X}_i \sim Normal(0,\sigma_v^2)$.

Given our data, the way we defined equation (13a) would lead us to work with positive values and, in conjunction with equation (13b), the model just described is a well-known generalization for the original standard Tobit. Note that differently from the traditional approach, the threshold is not constant along observations but vary in a deterministic way, say, $Prior_i = h(\mathbf{X}_i)$. Actually, the form of $h(\mathbf{X}_i)$ does not need to be specified, since we build our model conditional on $Prior_i$.

Finally, we just need to run a Tobit where the dependent variable is $(Prior_i - Post_i) \geq 0$ and the vector of regressors is $(Prior_i, \mathbf{X}_i)$. The estimated parameters are $(1 - \widehat{\gamma}, -\widehat{\delta})$. However, we need $(\widehat{\gamma}, \widehat{\delta})$ and interpret these results by means of equation (12a). Now refer to equations (12a) and (12b) and the interpretation is as follows:

i) Interpreting $\widehat{\gamma}$:

The fundamental result to obtain with respect to $\widehat{\gamma}$, attesting the validity of a Bayesian Update model, is its statistical significance. In other words, this tells us that *Prior* influences $Post^*$ and, thus, *Post*, i.e. we have a Bayesian effect in terms of Viscusi & O'Connor (1984).

If $\widehat{\gamma} > 1$, then an incremental change in $Prior_i$ induces a higher incremental change in $Post_i^*$. Hence, since equation (12b) is the minimun between them, we have $Post_i = Prior_i$, i.e. skepticism. If $\widehat{\gamma} < 1$, then an increase in $Prior_i$ induces a less than proportional increase in $Post_i^*$. Therefore, we are looking at the updaters, and $Post_i < Prior_i$.

ii) Interpreting $\widehat{\delta}$:

If $\widehat{\delta} > 0$, then an increase in $\mathbf{X}_i$ implies a higher $Post_i^*$ and we observe the same effect that $\widehat{\gamma} > 1$. Analogously, if $\widehat{\delta} < 0$, then lower $\mathbf{X}_i$ implies lower $Post_i^*$. Therefore, it is more likely that we observe an update.

In order to build the original Tobit likelihood function, we need the following expressions first:

$$P(Post_i = Prior_i | Prior_i, \mathbf{X}_i) = 1 - \Phi\left(\frac{(\gamma - 1)Prior_i + \mathbf{X}_i\delta}{\sigma_u}\right) \tag{14a}$$

$$P(Post_i = Post_i^* | Prior_i, \mathbf{X}_i) = \Phi\left(\frac{(\gamma - 1)Prior_i + \mathbf{X}_i\delta}{\sigma_u}\right) \tag{14b}$$

As well, we need the density $f(Post_i | Prior_i, \mathbf{X}_i, Post_i^* < Prior_i)$. But this is just $g(Post_i^* | Prior_i, \mathbf{X}_i, Update_i^* > Prior_i)$:

$$f(Post_i | Prior_i, \mathbf{X}_i, Post_i > Prior_i) = \frac{\phi\left(\frac{(Post_i - \gamma Prior_i - \mathbf{X}_i\delta)}{\sigma_u}\right)}{\sigma_u\left(1 - \Phi\left(\frac{(Post_i - \gamma Prior_i - \mathbf{X}_i\delta)}{\sigma_u}\right)\right)} \tag{15}$$

Defining the indicator function by $\mathbf{1}_{(\cdot)}$, the likelihood contribution of a given $Post_i | (Prior_i, \mathbf{X}_i)$ observation is:

$$\left[1 - \Phi\left(\frac{(\gamma - 1)Prior_i + \mathbf{X}_i\delta}{\sigma_u}\right)\right]^{\mathbf{1}_{(A)}} \times \left[\frac{\phi\left(\frac{(Post_i - \gamma Prior_i - \mathbf{X}_i\delta)}{\sigma_u}\right)}{\sigma_u\left(1 - \Phi\left(\frac{(Post_i - \gamma Prior_i - \mathbf{X}_i\delta)}{\sigma_u}\right)\right)}\right]^{\mathbf{1}_{(B)}}$$

where $A : Post_i = Prior_i$ and $B : Post_i > Prior_i$.

It is not difficult to show that the likelihood function of both Tobit models are equivalent, but this simple modification is the one overcoming our estimation problems. Also, the Tobit model represented by equations (13) can be straightforwardly implemented in any standard package as a plain censored regression with fixed (at 0) threshold. It is just necessary to represent the model as deviations around individuals' initial probability and interpret the estimated parameters accordingly.

4.3.2 Model 2: A Simple Two-Tiered Hurdle Model

An alternative modeling comes from the fact that we have a two-step structure. As mentioned before, individuals decide whether or not change their *priors* and then they give a *posterior* perception. Hence, *Discrete Choice Models* such as *Logit* and *Probit* followed by OLS on the updating subsample emerge as a natural approach. Thus, an alternative to the Tobit model is to dispense with the abstraction of a latent variable $Post_i^*$ and model only the observed empirical evidence directly. If we define $\Delta_i^{update} \equiv -Update = Prior_i - Post_i$[11] as the actual update to deal only with non-negative numbers, we have:

$$P(\Delta_i^{update} = 0 | Prior_i, \mathbf{X}_i) = 1 - \Phi(\gamma Prior_i + \mathbf{X}_i \delta) \tag{16a}$$

$$G(Post_i) | (Prior_i, \mathbf{X}_i, \Delta_i^{update} > 0) \sim Normal(\tilde{\gamma} Prior_i + \mathbf{X}_i \tilde{\delta}, \sigma^2) \tag{16b}$$

Equation (16a) dictates the probability that a real update has not occurred, i.e., $\Delta_i^{update} = 0$ and equation (16b) is a conditional model (conditioned on $\Delta_i^{update} > 0$) for the posterior probability. Equation (16b) just asserts that after a suitable transformation $G(Post_i)$, usually but not necessarily $G(Post_i) = \log(Post_i)$, we achieve normality. There are some features about the model just described:

i) We could have different explanatory variables for the censoring equation (16a) and equation (16b), as well as estimated parameters for the same explanatory can vary between equations;

ii) Estimation of $(\gamma, \delta, \tilde{\gamma}, \tilde{\delta}, \sigma^2)$ is quite simple, say: for equation (16a), run a probit; and for equation (16b) run a simple OLS for the subsample where $\Delta_i^{update} > 0$;

Estimation of partial effects demands knowledge of $G(\cdot)$. However, if we assume log-normality, $G(Post_i) = \log(Post_i)$, nice results emerge:

$$E(Post_i | Prior_i, \mathbf{X}_i, \Delta_i^{update} > 0) = \exp\left(\tilde{\gamma} Prior_i + \mathbf{X}_i \tilde{\delta} + \frac{\sigma^2}{2}\right)$$

$$E(Post_i | Prior_i, \mathbf{X}_i) = \Phi(\gamma Prior_i + \mathbf{X}_i \delta) \exp\left(\gamma Prior_i + \mathbf{X}_i \delta + \frac{\sigma^2}{2}\right)$$

An import drawback of the hurdle model is the impossibility of testing it against the Tobit specification, at least by simple procedures. On the other hand, a Vuong-type test could be applied to those non-nested hypotheses.

[11] Remember that in our data set $Prior_i \neq Post_i$, $\forall\ i$.

5 RESULTS

5.1 Bayesian Update with Skepticism: Model 1

For convenience, equations (12a), (12b), (13a) and (13b) are shown below:

$$Post_i^* = \gamma Prior_i + \mathbf{X}_i\delta + u_i \tag{12a}$$
$$Post_i = \min(Prior_i, Post_i^*) \tag{12b}$$
$$Prior_i - Post_i^* = (1-\gamma)Prior_i - \mathbf{X}_i\delta + v_i \tag{13a}$$
$$Prior_i - Post_i = \max(0, Prior_i - Post_i^*) \tag{13b}$$

Table (4) presents the estimation results. As explained in subsection (4.3.1), when referring to the latent equations (12a) and (12b), we see the likelihood of an update. This is the first decision step going on. Keep in mind that, for this part, we are interested in the coefficients of equation (12a) but, in fact, we estimate equation (13a). Thus, we need to proceed an adjustment on coefficients interpretation. For the $Prior_i$ coefficient, our interest relies on γ itself, but we would get an estimative of $(1-\gamma)$; therefore, it is easy to see that we must subtract it from 1. For all of the other coefficients, represented by δ, simply change estimatives' signs and, then, interpretation follows usual reasoning.

Table 4: Tobit Model

	Dependent variable:
	(Prior - Post)
Prior	0.502^{***}
	(0.129)
Age	-0.728^{***}
	(0.241)
Sex	-18.036^{**}
	(7.684)
Matching_Sex	63.480^{***}
	(13.931)
Educ_Int	19.307^{*}
	(11.459)
Constant	-232.149^{***}
	(39.056)
Observations	2,828
Log Likelihood	−1,017.116
Wald Test	41.278^{***} (df = 5)

*p<0.1; **p<0.05; ***p<0.01

Source: Elaborated by the author.

The first result we must obtain is the statistical significance of the *Prior* coefficient in equation (12a). Table (4) shows that $\hat{\gamma} = (1 - 0,5021) = 0.498$ and we do have a Bayesian effect. Also, note that $\hat{\gamma} < 1$, which implies that high *Prior*'s drive an update decision, a quite reasonable result. Remember that the official homicide rate is less than

1%, hence the higher initial responses are set, individuals are making higher mistakes, and the informational shock is strong enough to produce a change in perceptions.

The history behind **X** variables is straightforward. *Age* has an estimated coefficient equal to -0.728, i.e. $\hat{\delta} = 0.728$. Thus, respondents' age works undoubtedly against the update: older individuals are less likely to revise initial perceptions. The same direction is found in *Gender*. Being a female drops the likelihood of a change in perceptions. Perhaps both results tell us that vulnerability faced by older citizens and women make them more reluctant to update victimization expectations.

Despite much discussion on the importance of individuals' cognitive capability on probabilistic reasoning, our results show that, at least education is not relevant for this task. In fact, controlling for participants' level of education brought us insignificant coefficients. We argue that above a given threshold, ensuring a basic understanding of our probability explanation, the interviewer's perspicacity is crucial to persuade an update. This explains a 19.307 coefficient for the educational level of the informational sender. On average, graduates or undergraduates conducting our interview increase the likelihood of an update due to, we argue, a higher informational quality inherent to more educated interviewers.

Finally, the most important variable in our model is $Matching\_Gender$. Controlling by individuals' gender, if the interaction is composed by the same gender, i.e. Male/Male or Female/Female, the update is more likely to occur[12]. Clearly, due to reasons out of our scope in this study, the informational content depends heavily on gender correspondence. We conjectured in section (3) that gender is an important cultural issue in Latin countries like Brazil and, after careful econometric analysis, our results give evidence to support this idea.

5.2 Bayesian Update with Skepticism: Model 2

As proposed in subsection (4.3.2), we have a two-tiered model to deal with both updater and skeptical respondents. The first stage explains the changing decision with a *Logit* and a *Probit* model and then the second step applies OLS at the updating subsample. Unlike the previous subsection, the estimation results are interpreted directly. Table (5) and Table (8) present our results for the first and second stages, respectively.

Starting for the changing decision, it is important to highlight that our model is robust to the *Logit* or *Probit* choice. All explanatory variables are significant in the same magnitude with the same signs for both of them. Also, the Log Likelihood and the Akaike Information Criterion are almost the same. Although we can not say much on the coefficients magnitude, the signs are informative, and lead us to the conclusions made in the previous section. However, we must test our model adjustment.

In this sense, table (6) presents the prediction power of our models, where the variable *Predicted* is just our *Logit* and *Probit* probabilities estimation. We assume there is a threshold set at 50% such that if $Predicted \geq 50\%$ and $Change = 1$ or if $Predicted < 50\%$ and $Change = 0$, then we have a correct predicted choice. Finally, the variable *Correct* equals one if we had success in predicting and zero on the contrary. Therefore, *Correct* mean equals 0.957 implies that we had a 95.7% success rate.

Given our good adjustment, we proceed to interpret its coefficients. Table (7) presents the marginal effects of our explanatory variables over the changing decision probability.

[12]Due to the *ceteris paribus* assumption, we can not keep the *Gender* variable in the model and identify at the same time specific matchings such as Male/Female, Female/Male and so on.

Table 5: Changing Decision

	Dependent variable:	
	Change (*logistic*)	Change (*probit*)
Prior	0.007^{**}	0.003^{**}
	(0.003)	(0.002)
Age	-0.020^{***}	-0.009^{***}
	(0.007)	(0.003)
Gender	-0.498^{**}	-0.238^{**}
	(0.197)	(0.094)
Matching_Gender	1.876^{***}	0.804^{***}
	(0.410)	(0.162)
Educ_Int	0.654^{*}	0.261^{*}
	(0.337)	(0.141)
Constant	-5.879^{***}	-2.813^{***}
	(1.056)	(0.436)
Observations	2,829	2,829
Log Likelihood	−473.495	−473.564
Akaike Inf. Crit.	958.990	959.129

*p<0.1; **p<0.05; ***p<0.01

Source: Elaborated by the author.

With respect to the *Prior*, in accordance with our previous model, initial perceptions have a positive influence on the probability that an update occurs. High initial perceptions imply higher mistakes, hence, once more, this willingness to change can be seen as rational.

Also, for all of the other independent variables, the results lead to the same previous conclusions. In *Age*, as it was the case before, older participants are more reluctant to change and, again, perhaps insecurity drives this results. A stronger evidence of this explanation comes from *Gender*. Being female reduces the changing probability in 2 percentage points.

Table 6: Prediction Power

Statistic	N	Mean	St. Dev.	Min	Max
		Logit Model:			
Change	2,829	0.043	0.202	0	1
Predicted	2,829	0.043	0.027	0.002	0.147
Correct	2,829	0.957	0.202	0	1
		Probit Model:			
Change	2,829	0.043	0.202	0	1
Predicted	2,829	0.043	0.027	0.002	0.140
Correct	2,829	0.957	0.202	0	1

Source: Elaborated by the author.

As expected, educated informational senders had a positive impact on the changing decision. Here, clearness or even social aspects of education in a low-educated society might influence the information quality and credibility. Once again, homophily analysis arises. Being of the same gender as the interviewer is clearly the most important aspect in our data. Male respondents might consider reliable only information provided by male interviewers. On the other hand, female participants might feel more comfortable being interviewed by women. Many explanations are possible but, under this cultural context, it is intuitive that gender matchings reduce information noise and this is key to the informational content.

Table 7: Marginal Effects

	Dependent variable:	
	Change (*logistic*)	Change (*probit*)
Prior	0.0003^{**}	0.0003^{**}
	(0.0001)	(0.0001)
Age	-0.001^{***}	-0.001^{***}
	(0.0003)	(0.0002)
Gender	-0.020^{**}	-0.021^{**}
	(0.008)	(0.008)
Matching_Gender	0.075^{***}	0.070^{***}
	(0.014)	(0.013)
Educ_Int	0.026^{**}	0.023^{*}
	(0.013)	(0.012)
Constant	-0.236^{***}	-0.244^{***}
	(0.040)	(0.037)
Observations	2,829	2,829
Akaike Inf. Crit.	958.990	959.129

*p<0.1; **p<0.05; ***p<0.01

Source: Elaborated by the author.

Once having initial perceptions changed, now we proceed to the updating analysis. We have two key points to emphasize with this exercise: in line with our Tobit model and with Viscusi & O'Connor (1984), the *Prior* coefficient in a linear model is significant. This is a crucial evidence supporting our Bayesian approach both for the general and restricting cases. *Age* and *Gender* have exactly the same role as our previous analysis and there is no need to say anything else. In the same way, the interviewers' level of education has again an important role explaining posterior responses. As expected, since all *Post*'s are less than *Prior*'s, and these initial perceptions are more than 1000 times greater than the true value, more educated interviewers move participants closer to the truth. This explains a negative value for *Educ_Int* (-4.188).

The insignificance of $Matching\_Gender$ is new here. However, remember from table (3) that 93% of our updating subsample is composed by interactions of the same gender. Hence, perhaps we do not have variations enough to obtain a significant coefficient which means that this pre-evidence is just what we are looking for.

Also, given the flexibility of the two-tiered modeling, we could introduce another independent variable: *Police*. Its positive coefficient is just what one would expect. Higher levels of *Police* values indeed mean a worse evaluation. Therefore, it leads us to an intuitive result: posterior perceptions are higher or, in other words, the movement towards the truth is less strong, when police work is worse seen by respondents.

Table 8: Restricted Ols

	Dependent variable:
	Post
Prior	0.023^{***}
	(0.008)
Age	0.026^{*}
	(0.015)
Gender	0.988^{**}
	(0.455)
Matching_Gender	1.074
	(1.179)
Educ_Int	-4.188^{***}
	(0.975)
Police	0.494^{*}
	(0.273)
Constant	1.697
	(1.369)
Observations	121
Adjusted R^2	0.254
Residual Std. Error	2.375 (df = 114)
F Statistic	7.822^{***} (df = 6; 114)

$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Source: Elaborated by the author.

6 CONCLUSIONS

This dissertation was constructed under three major guidances: i) Individuals are rational decision makers when updating subjective perceptions; ii) There is an estimable linear regression model for the Bayesian Update process, a well-suited framework to deal with the revision of subjective perceptions; iii) There are other variables besides respondents' characteristics influencing this updating process.

With these ideas, we presented an initial sample of 4030 individuals regarding subjective risk perceptions about becoming a homicide victim for the following 12 months in Fortaleza, Brazil. Besides a remarkable size, much greater than many papers in the field, our data brought information about interviewers and matching aspects that were used to account for information quality. In addition, we had a very different data sample: 95% of respondents did not want to change their *Prior's*, setting $Prior_i = Post_i$. This made the non-updaters, or skeptical agents, our protagonists.

We showed that, under simple assumptions, Viscusi & O'Connor (1984) and Smith & Johnson (1988) could be extended at the same time. We proposed a multiple linear regression model in the context of a Bayesian Update approach using a vector of independent variables for respondents, interviewers and matching in gender. However, our empirical evidence imposed the development of a model able to accommodate both updaters and non-updaters.

Following this guidance, we proposed two alternative estimable models suited to a more general context than what is found in the literature. Indeed, we believe this generalization was the main achievement of this study. A modified Tobit, easily implemented in any statistical package, was developed as the first approach. The second model was a well-known two-tiered Hurdle model, allowing the possibility to use a different set of variables to explain the changing decision and the update.

Firstly, our results showed that we could proceed with a Bayesian Update approach, since our *Prior* coefficient was significant for every model used. More specifically, referring to our Tobit model, the initial response coefficient was $\hat{\gamma} = (1 - 0.5021) = 0.498$, with its p value less than 0.01. It permitted us to conclude that $Prior's$ induce an update through a latent variable. In other words, higher initial perceptions imply higher misleading perceptions and this leads to a change in responses. Despite the fact it is using a different framework, this first block is in accordance with what Viscusi & O'Connor (1984) presents, i.e. there is a statistically significant linear equation relating *priors* to *posteriors*.

Also, fundamentally, we could rationalize a non-updating decision following a perceived informational quality/credibility argument. This was made through respondents' age, gender and initial perceptions, as well as interviewers' level of education and matching in gender between members of the interaction.

We found that older participants and females are more reluctant not only to change initial responses, but also to choose the level of the new response, in case of an update. We argued that insecurity aspects might be used to explain these findings. Also, respondents' level of education was insignificant in our exercise. In fact, interviewers' level of education plays a key role in both the changing and updating magnitude decisions. This is a consistent piece of evidence supporting our main line of explanation: the perceived informational content. The relationship between sender's education and information quality is straightforward.

Finally, our results also raised strong evidence on homophily aspects. In almost every regression, $Matching\_Gender$ had a major impact on the decision to change and in the magnitude of the update. The only insignificant one was in a simple OLS on the updating subsample. However, in this group, 93% of the interactions happened under the same gender. It suggests that Latin cultures, as the Brazilian, do not pose weight on race questions, as it is the case for other cultures. However, it has an analogue restriction due, perhaps, to its characteristic machismo: gender.

In summary, these results tell us that heterogeneity on the informational sender side is equally important in an interaction. Above this intuitive idea, previous papers could not take this into account simply because either there was no change in senders, which is the case in Delavande (2008), or they were not able to track interviewers' heterogeneity and explore it as identification sources. Controlling both sides of the interview made possible to see clear patterns supporting an intuitive message: non-updaters are just as rational as uptaders. The major point to be considered is simply how informative the interaction was seen.

Further developments should be done in many aspects of this study. Firstly, the design of our interaction was clearly not ideal and the informational shock could be better customized to adjust official rates to different socioeconomic characteristics. Although it raised a remarkable opportunity to propose a generalization, our estimation procedure can be improved in several ways. One of particular importance is to allow different revision directions, since our data and modeling present just the downwards update. Also, we could have more information regarding interviewers. Further study on homophily is another gap to be filled.

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