

**A MAXIMUM COVERING MODELING FOR INTEGRATED VEHICLE SCHEDULING  
WITHOUT CHANGEOVERS****Bruno de Athayde Prata**

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**ABSTRACT**

Nowadays, the vehicle and crew scheduling is a hard, widely studied Combinatorial Optimization problem. Several studies have shown that exact approaches for solution of the above mentioned problem are prohibitive due to high computational cost involved. This paper describes a new approach for vehicle and crew scheduling without changeovers (the change of a vehicle of a driver are forbidden) based on a maximum covering problem formulation. Computational results with bus and crew scheduling problems from companies of Fortaleza, located in Northeast of Brazil, are presented and discussed. The results indicate the applicability of the proposed approach.

**KEYWORDS.** Vehicle and Crew Scheduling problem, mixed integer programming, maximum covering problem.

**Main area:** L & T - Logistics and Transport

## 1. Introduction

In mass transit systems, the planning of transport modes is composed of the following main stages (2001, CEDER, 2002): network route definition, setting of the timetables, vehicle scheduling, crew scheduling and crew rostering. The last three activities are the target of lots of research due to the potentiality of costs reduction coming from its optimization (DADUNA and PAIXÃO, 1995; WREN and ROSSEAU, 1995).

Several researchers have reported the strong interaction between the problems concerning vehicle and crew scheduling and the gain in considering them jointly (FRIBERG and HAASE, 1999; GAFFI and NONATO, 1999; FRELING *et al.*, 1999, FRELING *et al.*, 2003). Such problem is denominated Vehicle and Crew Scheduling Problem – VCSP. The mentioned papers also report the difficulty in solving the VCSP in an exact way, even for small size instances.

One of the cases in which the integrated approach has more advantages is the one in which the changes of drivers per vehicle (changeovers) are forbidden (FRELING *et al.*, 2003). For this case in particular, Huisman (2004) presents a model based on the Set Covering Problem – SCP.

This paper aims to report a new mathematical modeling for the VCSP without changeovers, based on the Maximum Covering Problem (MCP). The mathematical model proposed is tested in a set of 15 instances which come from the public transportation system in Fortaleza, a city which is located on Northeast of Brazil.

The paper is composed of four other sections, described as follows. In the second section, basic concepts about VCSP are presented. In the third section, a new mathematical model of VCSP is presented. In the fourth section, the computational experiments with real instances are presented. Finally, in the fifth section, conclusions about the theme approached are made.

## 2. Vehicle and Crew Scheduling

Traditionally, the vehicle and crew scheduling is accomplished in a sequential way, so that the vehicles scheduling (Vehicle Scheduling Problem – VSP) is an initial stage, being completed by the crew scheduling (Crew Scheduling Problem – CSP). CSP is more constrained than VSP; therefore, it is not very intelligent to schedule vehicles without taking the crews into account, since they constitute the bottleneck of the process.

A great deal of the public transportation companies, especially in the developed countries, the costs with the crew are the most significant ones in the service offered. Thus, efforts must be made to minimize such costs, always taking into consideration the vehicles allocation, which is also a fundamental question in such process.

Freling *et al.* (2003) affirm that there are three approaches for vehicle and crew scheduling: the traditional approach, in which the VSP is solved first, and then the CSP; the independent approach, in which both problems are solved separately, without the concern with the mutual impacts between the two schedules; and the integrated approach, in which the two problems are solved simultaneously. The focus given by the paper refers to the simultaneous resolution of the referred problems.

According to Freling *et al.* (1999), the benefits of the integrated approach for VCSP are bigger in the following circumstances: restrict number of changeovers, constrained deadhead trips, elevated time in which a crew is utilized in the vehicle, obligatory permanence of the crew while the vehicles are waiting, imposition of a minimal duration of pieces-of-work, domination of the crew costs over the vehicles costs and the rendition of the drivers occurs only in the depots.

In case any of these characteristics do not exist in the urban transportation planning, probably it will not be necessary to employ the integrated approach, since the crews can move independently from the vehicles. However, such situations are very usual in practice. It is also pertinent to highlight that the goals of the integration are two: to generate viable solutions and to reduce the system operation costs.

Taking into account the intrinsic complexity of the VCSP, for real instances of medium

and great size, it is not possible to obtain the precise solution for the problem. Thus, heuristic approaches were proposed in the literature for its solution.

Next, a bibliographical review of the approaches for the solution of the VCSP is presented. Due to the heterogeneity of such approaches, they were classified into two groups: approaches based on exact methods and approaches based on heuristics.

When it comes to the approaches based on exact methods, it is possible to refer to the proposals by Freling *et al.* (1999b), Friberg and Haase (1999), Gaffi and Nonato (1999), Haase *et al.* (2001), Freling *et al.* (2001b), Fischetti *et al.* (2001), Freling *et al.* (2003), Huisman (2004), Huisman *et al.* (2005), Weider (2007), Fleurent and Rosseau (2007), Groot and Huisman (2008) e Mesquita and Paia (2008).

Among the approaches based on heuristics, it is possible to highlight the approaches proposed by Ball *et al.* (1983), Falkner and Ryan (1992), Patrikalakis and Xerocostas, Wren and Gualda (1999), Valouxis and Housos (2002), Rodrigues *et al.* (2006), Laurent and Hao (2007), Bartodziej *et al.* (2007), Steinzen *et al.* (2007), Steinzen (2007) and Laurent and Hao (2008).

The use of optimization based on mathematical models has been infeasible for the VCSP, since even for instances with few trips (about 20 or 30 trips), the computational times become prohibitive. Such complexity consists of the fact of aiming at solving, in an integrated way, difficult problems such as the multicommodity network flow and the set partitioning problem/set covering problem. Thus, it is essential to search for a simpler mathematical model which even though is able to reflect appropriately the complexity of the VCSP.

### 3. Proposed approach

Huisman (2004) presents an extension of Freling *et al.* (1999) model for the VCSP, which generalizes this model, being applied when changeovers are allowed or not. However, Huisman (2004) underlines that, when changeovers are forbidden, the concept of combined duty can be used, in which a same vehicle and crew duty can be combined in only one duty. Thus, the cost of a combined duty  $j$ , denoted  $g_j$ , is defined as the addition of the duty cost of the vehicle plus the cost inherent to the crew duty.

In set covering and set partitioning problems, there is a matrix, usually denoted by  $A$ , which consists of  $n$  rows and  $m$  columns. In these problems, the aim is to cover the highest number of  $A$  rows with the lowest cost possible. In planning problems concerning public transportation, the  $A$  columns represent duties and the rows represent pieces-of-work.

The set covering problem, although is solved easier than the set partitioning problem, it is not the desirable approach for the CSP, since, generally, pieces-of-work are covered for more than one duty, which is an undesirable situation in practice. Thus, a stage of post-processing of the solution aiming at eliminating the superposition of duties is necessary.

Dias *et al.* (2002) present a model for the Bus Driver Scheduling Problem – BDSP based on the Relaxed Set Partitioning Problem – RSPP. On RSPP, leftovers (associated at the uncovering of all the pieces-of-work) are allowed through a relaxation of the classic sets partitioning problem. However, such uncovering is penalized at the objective function.

According to Park and Ryu (2006), instead of adopting a SCP model for the optimization of duties related to the crew, a Maximal Covering Problem – MCP can be used to find a subset of duties which cover the biggest quantity of pieces-of-work possible. According to the experience of Park and Ryu (2006), the MCP approach lets fewer pieces-of-work uncovered than a SCP model. On this approach a standard which acts as a superior limit of duties used in the solution is used.

The Maximum Covering Problem (MCP) is a widely studied Combinatorial Optimization problem, with several applications, such as facility location (including health centers, emergency vehicles and commercial bank branches) and scheduling (flexible manufacturing systems, mass transit services, telecommunications) (CHURCH and ReVELLE, 1974; DOWNS and CAMM, 1996; RESENDE, 1998; ARAKAKI and LORENA, 2006; PARK and RYU, 2006).

The classic MCP is defined by Church and ReVelle (1974) as a maximization problem,

in which weights are associated to the decision variables. Church and ReVelle (1974) formulate the MCP as a problem of minimization, associating it to a  $c_i$ , cost, inherent to the uncovering of the  $i^{\text{th}}$  line in  $A$ . Thus, the goal is to minimize the number of uncovered lines in matrix  $A$  (such formulation could be denoted as a problem of minimum uncovering).

According to Dias *et al.* (2002), in problems concerning operational planning of public transportation systems, approaches based on set covering and set partitioning are frequently utilized.

Approaches based on SCP have the advantage of enabling a faster solution; however, due to the permission of overcovers occurrence, such solutions need corrections to be put into practice. Approaches based on SPP are of harder solution, however, since they do not allow overcovers, they are more attractive in a practical point of view.

Huisman (2004) underlines that, for the particular case of VCSP in which changeovers are not allowed, an approach based on SCP can be adopted. As a consequence, an approach based on SPP can also be utilized. As the model based on the set partitioning problem is more desirable in a practical point of view, it is pertinent to analyze it thoroughly.

According to Klabjan *et al.* (2001), the main reasons why the problem of crew scheduling based on SPP makes it difficult are: the high number of feasible duties required for cover all trips, its complex structure and the complex nature of non-linear costs.

The number of possible duties is not basically related to the mathematical model, but to the practical characteristics of a given real problem.

One of the main reasons why the SPP structure is complex is the equality constraints, which impose that every line of matrix  $A$  be covered by only one column. However, this is the great advantage of the abovementioned approach because, thus, the overcovers are eliminated.

The objective function of SPP consists of an inner product of a costs vector and a decision variable vector. So, the problem has an explicit structure of costs with a linear feature. On the other hand, in an implicit way, this structure might change into a non-linear one. Such particularity makes itself evident in problems of the crew scheduling kind.

In problems concerning public transportation planning, a duty can be seen as a covering of a set of pieces-of-work. The cost of a duty is related to the payment of the crew. Due to the labor laws valid in most of the cities, the crew duties are not paid for each hour of work, but for workday. Thus, duties that cover quantities of different pieces-of-work might have the same cost. Consequently, the relation between the cost of the duty and the quality of the duties is not directly proportional. In other words, such relation is non-linear.

Another source of non-linearity of the implicit SPP costs structure is the payment of extra-hours. Once exceeded the duration of the workday, so that a duty can cover more pieces-of-work, an addition of an extra amount to be paid is necessary, which is proportionally more expensive than an hour worked in a normal workday. There is, again, an evidence of the non-linearity of SSP costs.

A way to reduce the high number of feasible SPP duties, as well as to avoid its nature of non-linear costs is working with an objective function which has as a priority not the minimization of costs of the duties to be selected, but with the minimization of uncovered pieces-of-work.

Proceeding this way, we can generate only duties within the standard workday. Thus, the objective function would related to the minimization of the amount of pieces-of-work to be covered by extra-hours. Another advantage of this approach would be the reduction of the amount of duties to be generated, since the duties with extra-hours would not be necessary.

A new variant of the MCP, applied to the VCSP without changeovers, can be mathematically formulated as follows:

$$\text{Minimize} \quad z = \sum_{i=1}^n z_i \quad (1)$$

Subject to:

$$\sum_{i=1}^n a_{ij}x_j + z_i = 1 \quad \forall i = 1, \dots, n, \forall j = 1, \dots, m. \quad (2)$$

$$\sum_{j=1}^m x_j \leq d \quad \forall j = 1, \dots, m. \quad (3)$$

$$x_j \in \{0,1\} \quad \forall j = 1, \dots, m. \quad (4)$$

$$z_i \geq 0 \quad \forall i = 1, \dots, n. \quad (5)$$

The objective function represented by equation (1) has as a priority to minimize the number of uncovered pieces-of-work. In case the  $i^{\text{th}}$  piece-of-work cannot be covered, the set of constraints of type (2) imposes that  $z_i = 1$ . As a consequence of this constraint, we have the integrality of the variables;  $z_i$  can be relaxed. Constraint (3) imposes that a maximum number of  $d$  columns of matrix  $A$  be selected in the solution. The set of constraints of type (4) illustrates the binary feature of the decision variables. The set of constraints of type (5) imposes the non-negativity of variables  $z_i$ . The variable  $x_j = 1$ , if the duty  $j$  is in the solution, being 0 otherwise.

With relation to the formulation proposed above, the following comments can be highlighted: (i) The variables of type  $z$  can be considered auxiliary variables, since, in fact, they do are not effectively characterized as decisions. If  $z_i = 1$ , then a leftover occurs. (ii) The set of constraints of type (2) forbids the occurrence of leftovers, distinguishing itself from the traditional formulations for the MCP by adopting equality constraints. (iii) In opposition to the classical models of maximum covering, in which the use of  $d$  columns is imposed, constraint (3) allows that the number of columns inferior to  $d$  be utilized. (iv) The linear relaxation of the constraint set of kind (4) incurs in the obtainment of a lower bound for the solution of the model.

#### 4. Case study

The city of Fortaleza, capital of Ceará, is one of the regional Brazilian metropolises in crescent expansion. It has about 2,4 millions of habitants, distributed in just two thousands of Ceará's territory, constituting almost one third of the State's population. It is the first capital of the country in terms of population density. Taking into account its metropolitan area, the population of Fortaleza is superior to 3,2 millions of habitants.

In Fortaleza, employment is concentrated mainly on the central area of the city, and most part of the trips is accomplished by the bus transport system. Most part of the population has a low income, and they depend on the bus system to travel, to study, to shop and to travel for leisure.

The public transportation system of Fortaleza consists of a complex network of transportation, including a range of lines and transshipment terminals. In spite of this complexity, the vehicle and crew scheduling is still done manually, which incurs to a big period of planning time and in solutions which quality could be improved.

The main characteristics of the vehicle scheduling in Fortaleza are: (i) the bus scheduling is a responsibility of the Urban Transportation Enterprise of Fortaleza – ETUFOR, public and private enterprise that manages public transportation like buses, vans, taxis and motorcycle-taxis in the city; (ii) the bus scheduling is planned for each line, individually; (iii) exchanges of vehicles are not allowed between lines during the breaks; (iv) each line is operated by a set of enterprises; (v) the times of pull-ins and pull-outs are not considered in the scheduling, once you do not know, at first, which enterprises are going to operate a determinate line; and (vi) each vehicle has fuel autonomy to make the journey.

There are 22 enterprises operating in the city, with a fleet of 1700 vehicles approximately and a total of 221 regular lines and 22 lines of special operation (dawn buses). There are still 16 lines of complementary transportation (vans).

Among the characteristics of the crew scheduling in Fortaleza, it would be interesting to emphasize: (i) the crew scheduling is not the manager organ's responsibility (ETUFOR); however, the vehicle scheduling must respect the labor legislation appropriate to the crews; (ii)

the crew of a bus is formed by a driver and a collector; (iii) the workday lasts 07h20min which is the minimum remunerated value to the crew, being allowed a maximum workday duration of 09h20min, with the totaling of extra hours that are paid with an addition of 50%; (iv) a break is obligatory (unremunerated) for meal that must have a minimal duration of 30min. The meal break should last from 30min to 1h. Breaks of 1h or 2h are allowed, but they generate complaints among the crew; (v) according to an agreement with the workers, a maximum value of 30% of the trips can have breaks bigger than 2h; (vi) the meal break should happen out of the rush hours, preferentially between 9:00 and 11:00, during the morning period, and between 14:30 and 16:30, during the afternoon period; and (vii) a meal break should not be conceded to two vehicles that operate forthwith in a same line.

It is imposed to a crew to work in the same vehicle during the workday, therefore, changeovers are not allowed.

Another pertinent matter is the intrinsic relationship between the vehicle and crew scheduling in Fortaleza, since changes of the crew are not allowed among vehicles. This way, it makes little practical sense to schedule the vehicles without taking into consideration the duties of the crew, which makes an integrate approach indispensable to the problem.

According to Huisman (2004), as the changeovers are forbidden, it is considered that every trip constitutes a piece-of-work. This way, a vehicle-crew pair is allocated to every piece-of-work, in a way that all of the trips are covered with the smaller number of vehicles possible and that a smaller quantity of remunerated hours to the crew is required.

Having in mind the possibility of utilization of duties that violate the labor constraints, the generic model proposed in section 3 can be adapted to the study case as follows.

A matrix  $A$  being given, of  $n \times m$  order, with  $a_{ij} \in \{0,1\}$  elements; a matrix  $B$ , of  $n \times p$  order, with  $b_{ik} \in \{0,1\}$  elements; a binary decision variable of  $x_j$  that equals to one if the  $j^{\text{th}}$  duty of the matrix  $A$  is in the solution, being zero in otherwise;  $y_k$  a binary decision variable that equals to one if the  $k^{\text{th}}$  duty of the matrix  $B$  is in the solution, being zero otherwise;  $z_i$  a binary decision variable that equals to one if the  $i^{\text{th}}$  trip (piece-of-work) is not covered, being zero otherwise and  $\gamma$  the maximum number of duties allowed from matrix  $B$ , the mathematical model for the VCSP without changeovers, based on MCP, is defined as follows:

$$\text{Minimize} \quad \sum_{i=1}^n z_i \quad (6)$$

Subject to:

$$\sum_{j=1}^m x_j + \sum_{k=1}^p y_k \leq d \quad \forall j=1, \dots, m. \quad (7)$$

$$\sum_{i=1}^n a_{ij} x_j + \sum_{k=1}^p b_{ik} y_k + z_i = 1 \quad \forall i=1, \dots, n, \forall j=1, \dots, m; \forall k=1, \dots, p. \quad (8)$$

$$\sum_{k=1}^p b_{ik} y_k \leq \gamma \quad \forall k=1, \dots, p. \quad (9)$$

$$x_j \in \{0,1\} \quad \forall j=1, \dots, m. \quad (10)$$

$$y_k \in \{0,1\} \quad \forall k=1, \dots, p. \quad (11)$$

$$z_i \geq 0 \quad \forall i=1, \dots, n. \quad (12)$$

The objective function represented by the equation (6) intends to minimize the number of uncovered pieces-of-work. In case  $i^{\text{th}}$  piece-of-work cannot be covered,  $z_i = 1$ . Constraint (7) imposes that a maximum number  $d$  of the matrix columns  $A$  and  $B$  be selected in the solution. The set of constraints of the type (8) assures that the pieces-of-work will be covered by columns with no intersections of rows in distinct columns, and, in case this is not possible,  $z_i=1$ . The constraint of type (9) imposes that the maximum quantity of unfeasible duties is not exceeded.

An unfeasible duty is a duty that does not attend completely the labor legislation. The expressions (10), (11) and (12) impose the character of the variables. It must be observed that the variables  $z_i$  are auxiliary, but they do not represent, in fact, decisions. The variable  $x_j = 1$ , if the duty  $j$  is in the solution, being 0 otherwise.

The matrix  $A$ , which has  $n$  rows and  $m$  columns, contains feasible duties that respect the labor legislation, while matrix  $B$  that has  $n$  rows and  $p$  columns, contains unfeasible duties. It is important to bring to light that there is no penalization in not covering a line in matrix  $B$ , because this kind of duty is not priority.

The logic to the vehicle and crew scheduling used was the following: (i) generation of possible duties (feasible and unfeasible); (ii) pre-processing; e (iii) resolution of the mathematic model. Hereafter, these stages will be concisely described. The duty generation stage is fundamentally important to the quality of the solutions generated in the optimization process. The search technique in graphs known as Depth-First Search is usually used to generate all feasible duties. It is from the set of these duties, generated in a very high number, that the duties are afterwards selected to implement practically.

However, many works (KLABJAN *et al.*, 2001; KORNILAKIS and STAMATOPOULOS, 2002; AHMADBEYGI and COHN, 2006) report that the generation of all feasible duties can hamper or even make unfeasible the optimization process, having in mind that the space of feasible solutions grows exponentially with the quantity of generated duties.

Ahead the fact that the vehicle scheduling in Fortaleza requires a generation of a set of unfeasible duties, an itemizer process, as the Depth-First Search, it could incur in prohibitive computer costs in the optimization process. Therefore, it was taken as an option to generate duties heuristically, according to the procedure described as follows: the algorithm seeks to generate duties with a meal break, allocating the other trips with the smallest break possible (if possible, with no break). This way, the heuristic seeks to generate good duties. The matrix  $A$  was generated with breaks of [30;60], while matrix  $B$  was generated with breaks of more than 60 minutes. With the stop of the algorithm criterion, it was used a maximum number of iterations  $w$  equal to  $20n$  (in which  $n$  is the number of trips), because, this way, some thousands of duties would be already generated to the optimization stage.

Duties with extra hours were not generated, only duties with a maximum duration of 07h20min, due to the definition of the model described in Section 3 in this current paper.

The pre-processing stage consists in the transformation of the generated duties in a file to be run in the optimization solver. The heuristic and the pre-processor were developed in the integrated environment Lazarus ([www.lazarus.freepascal.org](http://www.lazarus.freepascal.org)), IDE of public domain that works with Pascal language. The solver used was LINGO ([www.lindo.com](http://www.lindo.com)), optimization *software* used to solve the problems in the linear, integer and non-linear programming. The computational experiments were made in a Genuine Intel 1.86 GHz, with 1GB RAM.

For the evaluation of the proposed approach, it were analyzed 15 instances, reported in Table 1, which consist in about 5% of the bus lines in the mass transit system in Fortaleza.

The Table 1 is composed by thirteen columns, described hereafter. Column 1 consists in the identification of the instance, which is represented by the number of the line. Column 2 presents the number of trips of the line. In columns 3 and 4 the quantity of duties generated are presented. In column 5 it is illustrated the cycle time of the line. It must be observed that in the public transportation system by bus in Fortaleza the time of the cycle varies during the day. Therefore, the mode of cycle time is presented. In column 6 is presented the maximum number of columns to be taken into the solution. In columns 6 and 7 the densities of the matrixes are presented. The column 9 presents the value of the optimized solution to the instance, while in column 10 the computational time is presented to the acquisition of this solution. In columns 11 and 12 are exposed Branch-and-Bound results. Finally, in the column 13 is presented the covering percentage.

The quality of a solution to MCP is normally measured through the covering percentage  $c$  (column nine in Table 1), determined by the following expression, in which  $n$  is the number of pieces-of-work and  $z$  is the value of the objective function obtained, which represents the number

of pieces-of-work uncovered, which must be covered by extra-hours duties.

$$c = \frac{(n - z)}{n} \quad (13)$$

The value of  $d$  is given by the minimum quantity of duties required to cover the trips in a line, which can be calculated multiplying the cycle time by the number of trips and dividing this result by the duration of the labor journey. The value of  $\gamma$  is given by  $\lfloor 0,3 \times m \rfloor$ . It was agreed that the number of unfeasible duties would be equal to the number of feasible duties.

In the matter of the obtained results, the following conclusions can be emphasized: (i) to the 15 comparisons made, the mathematic model incurred in an improvement of the covering factor in 11 instances, and obtained results inferior to the manually solutions in 3 instances and occurred 1 tie; (ii) the gap of the covering percentage is found on the interval  $[-3,1;14,7]$ , while the average gap of the covering percentage for the 15 instances was of 4,7%.

**Table 1. Computational results.**

1. Instance	2. $n$	3. $M$	4. $p$	5. Cycle (min)	6. $d$	7. $\rho_A$	8. $\rho_B$	9. ZIP	10. $t$ (s)	11. Number of branches	12. Number of iterations	13. $c$ (%)
11	179	1005	1079	65	30	3,2	3,1	3	10	124	39718	98,3
466	74	509	509	56	10	9,2	9	4	1	89	5089	94,6
112	109	657	657	60	16	5,5	5,4	2	20	610	131278	98,2
407	67	247	247	90	20	5,5	5,4	4	1	38	2755	94,0
340	75	627	627	40	8	11,1	10,8	4	2	2	2793	94,7
360	214	1301	1301	60	32	2,8	2,8	7	476	14543	2225640	96,7
220	142	1004	1004	55	20	5,0	4,9	6	11	465	74471	95,8
501	64	489	503	48	8	12,2	12	3	1	16	1644	95,3
401	166	1164	1164	52	24	4,2	4,1	7	158	2680	858401	95,8
810	64	489	489	48	8	11,9	11,6	3	4	88	10945	95,3
905	49	258	258	72	8	21,2	10,0	1	1	0	427	98,0
15	111	801	804	56	14	6,5	6,4	4	106	3007	560602	96,4
122	64	440	440	52	8	10,7	10,5	4	20	635	204087	93,8
314	74	325	325	77	14	5,9	5,9	4	1	3	1085	94,6
70	83	295	302	100	22	4,4	4,3	1	1	4	900	98,8

From the results exposed on Table 1, the following conclusions can be taken into consideration: (i) the covering value percentage is found on the interval  $[93,8;98,8]$ , while the average covering percentage to the 15 instances was of 96,0%; and (ii) in 12 instances, computational time was inferior to one minute.

From the obtained results, it can be deduced that the biggest benefit in the developed approach (with base on a new mathematic model to the problem) is a reduction of the planning time. While the generation of vehicle schedules generated manually can take hours or even days to be done by the technicians of the ETUFOR, the proposed approach allows that the scheduling is done rapidly, making a greater flexibility in the scheduling by the technicians of the enterprise possible.

These time savings have an important additional effect, since they allow the realization of multiple trials in a reduced work period, with the possibility of an analysis of the sensitivity of the proposed solutions.

The comparison among the results provided by the model and the manually generated solutions was made, but is not completely fair. This because the manually generated solutions flex the labor constraints even more (for example, they allow breaks with less than 30 minutes or journeys superior to 9:20hs), a fact that is not allowed. Anyway, even flexing these constraints, the manually generated solutions many times require a lot of extra hours duties to cover the trips.

In Table 2 is illustrated a comparison among the generated solutions by the mathematic model and the manually generated solutions, in terms of pieces-of-work covered and covering percentage.

**Table 2. Comparison between optimal and manual solutions.**

Instance	Trips	Uncovered pieces-of-work		Covering (%)		Gap (%)
		Model	Manual	Model	Manual	
11	179	3	5	98,3	97,2	1,1
466	74	4	12	94,6	83,8	10,8
112	109	2	4	98,2	96,3	1,8
407	67	4	4	94,0	94,0	0,0
340	75	4	6	94,7	92,0	2,7
360	214	7	30	96,7	86,0	10,7
220	142	6	17	95,8	88,0	7,7
501	64	3	1	95,3	98,4	-3,1
401	166	7	6	95,8	96,4	-0,6
810	64	3	1	95,3	98,4	-3,1
905	49	1	8	98,0	83,7	14,3
15	111	4	8	96,4	92,8	3,6
122	64	4	10	93,8	84,4	9,4
314	74	4	5	94,6	93,2	1,4
70	83	1	12	98,8	85,5	13,3

## 5. Conclusions

This paper presented a new approach for the Vehicle and Crew Scheduling Problem in public transportation systems based on a maximum covering model. This formulation showed itself consistent and allowed the solution, in a exact way, of 15 instances originated from the public transportation system by bus of Fortaleza.

It is indispensable to emphasize the relevance and the originality of the approach here proposed. There is no paper that reports a formulation for the Vehicle and Crew Scheduling Problem based on the MCP, even on the best knowledge of the authors.

To the authors, the obtained results are of high quality, which show the validity of the proposed approach and its potentiality of application in other real problems as well.

This paper reports just a part of a wide research project. Having in mind the multiobjective nature of the vehicle and crew scheduling problems in public transportation systems, the authors are developing models and algorithms that take into consideration the multicriteria nature of the problem.

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