VISCOELASTIC AND ELASTIC STRUCTURAL ANALYSIS OF FLEXIBLE PAVEMENTS

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Abstract. *Hot Mix Asphalt layers have been typically considered with linear elastic behavior, with the elastic modulus determined in repeated load tests performed under diametral compression (resilient modulus). However, it is well known that asphalt mixtures have a viscoelastic behavior. The effect of elastic on viscoelastic behavior in the structural response is significant for paviment design. In this work, numerical analysis provides information about three parameters used in pavement design: (i) displacement on the top of the asphalt surface layer; (ii) tensile stress at the bottom of the asphalt surface layer and (iii) compression stress at the top of subgrade granular layer. The asphalt layer is considered either elastic or viscoelastic in each analysis performed. Considerations about the load time durations are also presented.*

Keywords: *Pavement analysis, Viscoelasticity, Finite elements, Asphalt*

1. INTRODUCTION

Flexible Pavements are multilayered systems composed by an asphalt mixture as the surface layer and a few number of granular layers. In the calculation of stresses and strains in multylayered systems can be used analytical solutions based in the Theory of the Elasticity for the simplest cases. In the most complex cases, programs based on numeric methods, mainly the Finite Element Method (MEF), were developed as approximate solutions. An advantage of numerical approach is the use of various constitutive models, such as, linear elastic, nonlinear elastic, elasto-plastic, viscoelastic, viscoplastic to describe the behavior of several materials. Pavement design tries to prevent the major distress such as permanent deformation (rutting) and fatigue cracking.

Since the early 1960s, pavement engineers have gone from empirical to mechanistic-empirical methods for pavement design (Brown, 1977; Huang, 1993; Medina, 1997). The latter is basically divided in two parts: (i) a mechanistic part related to the prediction of stresses, strains and deflections in the pavement layers due to mechanical load on the pavement surface. Herein, fundamental physical properties are used with a theoretical or numerical model; (ii) an empirical part of the method relates the calculated structural response to cracking and permanent deformation by means of performance models. For instance, fatigue cracking is commonly predicted from the horizontal strain in the Hot Mix Asphalt (HMA) layer, while rutting is predicted from the maximum compressive stress on top of subgrade (Loulizi *et al*., 2002).

This paper presents a structural analysis of a pavement considering the viscoelastic behavior of the asphalt mixture. It is shown the time and rate dependency by means of the application of different load pulses. The analysis is also performed considering the elastic properties of the asphalt layer in order to compare with the viscoelastic behavior. The responses considered are: (i) deflection on the top of the asphalt surface layer, (ii) tensile stress at the bottom of the surface layer and (iii) vertical stress at the top of the granular subgrade.

2. BACKGROUND

2.1 Theoretical and computational analysis of flexible pavements

Burmister's analytical responses for elastic multilayered systems (Burmister, 1943,1945) are used for pavement analysis (Yoder and Witczakc, 1975; Huang, 1993). It is usually assumed static loading, continuity conditions at the interfaces between layers, homogeneous, isotropic, and linear elastic materials. With these assumptions, only two material properties are necessary: (i) Modulus of Elasticity (E) and (ii) Poisson's ratio (υ). In pavement design and analysis, the Resilient Modulus (RM) is frequently used as E and is based on the recoverable strain under repeated loads measured in the laboratory using a haversine stress pulse when testing HMA specimens. Poisson's ratio is usually assumed 0.3. This layered-elastic closed solutions were implemented a multilayer elastic programs. Ahlborn (1972), in the University of California at Berkeley, developed a widely used program, ELSYM, today ELSYM5 (FHWA, 1985). Despite its limitations on the material constitutive modeling, ELSYM5 allows for a realistic representation of the field load since it accepts more than one loaded area.

Given the complexity of the boundary conditions, specific constitutive models of pavement materials as well as the improvement of computational methods have been used to determine pavement response. Researchers such as Duncan *et al*. (1968) started using the Finite Element Method (FEM) in the pavement structural analyses. FEM has some advantages over layered elastic solutions because it provides greater flexibility in modeling the nonlinear response characteristics of all the materials that make up the pavement section (Monismith, 1992). The works of Dehlen (1969) and Hicks (1970) show the importance of non-linear behavior of granular materials in the final pavement response. The program ILLIPAVE (Thompson and Elliot, 1985) incorporated many of the developments of Duncan *et al*. (1968) using resilient models (Dehlen, 1969; Hicks, 1970). In Brazil, the most widely used for pavement analyses program is FEPAVE2 (Motta, 1991). This program, originally developed in Berkeley and changed over the years in Brazil (Silva, 1995), considers the asphalt layer as linear elastic, sometimes the RM is considered a function of temperature, and the sublayers as nonlinear elastic (Duncan *et al*., 1968; Hicks, 1970). Given that the program only has axisymmetric elements, the load is typically considered circular with a radius of 10.8 cm and a tire pressure of 0.56 MPa.

It is well known that asphalt mixtures have a viscoelastic behavior (Goodrich, 1991; Pinto, 1991; SHRP, 1994; Lee and Kim, 1998; Daniel and Kim, 2002; Soares and Souza, 2003). Thus, its mechanical response exhibits time and rate dependency and the consideration of its behavior as elastic is not realistic. Structural pavement responses, such as stresses and strains, can be more accurately predicted by the consideration of the viscoelastic nature of the asphalt mixture.

Among of the viscoelastic programs applied to pavement analysis, the program VEROAD (Hopman, 1997). It is a linear viscoelastic multilayer program that accounts for the viscoelasticity of the asphalt material and the movement of the wheel using Fourier transforms. Its disadvantage is that the viscoelastic material is modeled just by the Burger's model (only four parameters). KENLAYER (Huang, 1993), also widely used, is based in quasi-elastic solutions by the collocation method (Schapery, 1961). Neither of them uses FEM and consequently there is no flexibility in some applied boundary conditions.

Brown (1973) derived an equation to calculate the loading time as a function of speed and depth beneath the pavement surface. The loading time was considered as the average of the stress pulse duration in the three directions as obtained from the elastic layered theory. The relation between loading time t (s), depth d (m), and vehicle speed v (km/h), is as follows:

$$\log(t) = 0.5d - 0.2 - 0.94\log(v) \tag{1}$$

The loading time as defined in Eq. 1 is equal to the inverse of the angular frequency of the applied sinusoidal wave. Barksdale (1973), on the other hand, defines the loading time as the duration of the sinusoidal or triangular pulse. Brown showed that his loading time is equal to 0.48 times the loading time as defined by Barksdale. McLean (1974) developed a chart to determine the pulse width of an applied square wave as a function of vehicle speed and depth underneath the pavement surface. The pulse duration of the square wave is shorter than that of the triangular or sinusoidal pulse.

2.2 Viscoelasticity of bituminous materials

In order to allow an analysis of the behavior of asphalt mixtures, it is essencial the determination of the viscoelastic properties. Viscoelastic materials are characterized commonly under (i) harmonic load or (ii) constant static load. In the first case, the material is characterized by the complex module (G^*) and the phase angle (δ). Under constant static load, the viscoelastic behavior is characterized through the creep compliance or the relaxation modulus given by Eq. 2 and Eq. 3, respectively:

$$D(t) = \frac{\varepsilon(t)}{\sigma_0} \tag{2}$$

$$E(t) = \frac{\sigma(t)}{\varepsilon_0} \tag{3}$$

where, D(t): creep compliance; $\varepsilon(t)$: strain as a function of time; σ_0: applied constant applied stress; E(t): relaxation modulus; σ (t): stress as a function of time and ε_0: applied constant applied deformation.

As the viscoelastic materials present time- and rate- dependent behavior, their response does not depend only on the applied load (or displacement) in the a specific instant, but of the whole load (or displacement) history (Christensen, 1982). In the case of uniaxial state, the stress-strain relationship is given under the form of convolution integrals, with the following stress formulation:

$$\varepsilon(t) = \int_0^t D(t-\tau)\frac{\partial\sigma}{\partial\tau}\partial\tau \tag{4}$$

and the following strain formulation:

$$\sigma(t) = \int_0^t E(t-\tau)\frac{\partial\varepsilon}{\partial\tau}\partial\tau \tag{5}$$

where, *t*: time starting from any referential, and time starting from the beginning of the application of the load.

Starting from Eq. 4, one can obtain the strain response of a linear viscoelastic material once it is known the stress history and the creep compliance. In a similar way, one can determine the stress response of a linear viscoelastic material submitted to a known strain state by knowing the relaxation modulus of the materials (Eq 5).

The characterization of the viscoelastic materials can be done by mechanical models (Huang, 1993). For the Voigt model, the creep compliance takes the form of Eq. 6, known as Prony series, which is one of the mathematical representations commonly used in the representation of viscoelastic solids (Lakes, 1999).

$$D(t) = D_0 + \sum_{i=1}^{N} D_i\left[1-\exp\left(\frac{-t}{T_i}\right)\right] \tag{6}$$

where, D_0 and D_i's are coefficients of the Prony series; t is the time of application of the load; T_i is the relaxation time ($T_i = \eta_i / E_i$), in which η_i is the viscosity and E_i is the modulus of elasticity. The methods more commonly used in obtaining these parameters (D_0, D_i's e T_i's), are the method of the successive residues and the collocation method (Schapery, 1961). Both use the creep results in the determination of the coefficients of the Prony series (Souza and Soares, 2002).

The time depedent model used herein to simulate the stress-strain relationship in asphalt mixtures is a Hooke model (spring) connected in series with a group of Kelvin models (dashpot in parallel with spring) also in series as depicted in Fig. 1.

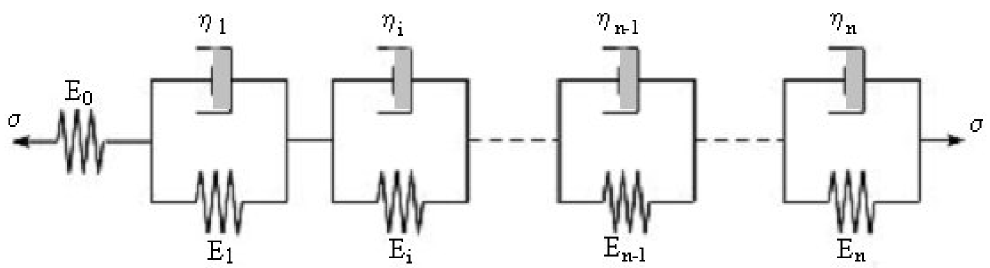


Figure 1 – Generalized Voigt's model

One way to incorporate the viscoelastic constitutive relations into a finite element formulation is incrementing these relations with respect to time. The formulation used in the present study is the one proposed by Zocher *et al*. (1997), and it is based on the incrementalization of Eq. (5), with the strain rate variation assumed to be constant during each time increment, and the relaxation modulus $E(t)$ represented by a Prony series (Wiechert model or generalized Maxwell model), as indicated in Eq. (7).

$$E(t) = E_\infty + \sum_{i=1}^{n} E_i \cdot e^{\frac{-t}{\rho_i}} \tag{7}$$

where:
E_∞, E_i and ρ_i: coeficients of the Prony series and n: number of terms.

This algorithm was used in several works (Foulk et al., 2000; Searcy, 2004, Souza et al., 2004; Evangelista-Junior et al., 2005) in structured language FORTRAN for Constant Strain Triangle (CST), Brick8 and Q4 linear elements. In the present work, a new finite element system was implemented in C++ and using the Object Oriented Methodology. This program considers both elastic and viscoelastic materials, different analyses models (plane stress, plane strain, axisymmetric and three-dimensional solid), as well as different element shapes (triangular, quadrilateral, bricks) and interpolation order (linear and quadratic).

3. MODELING

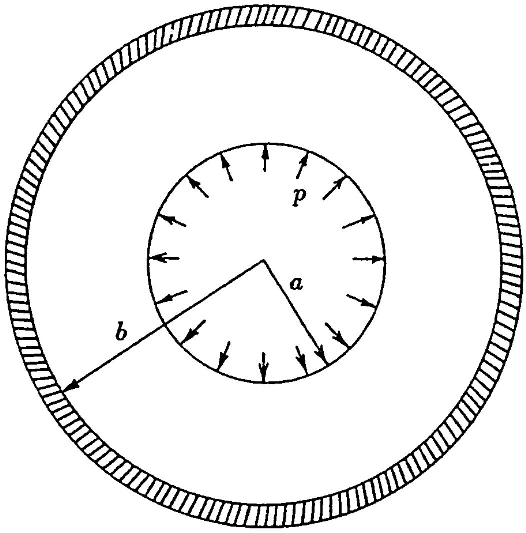
3.1 Validation

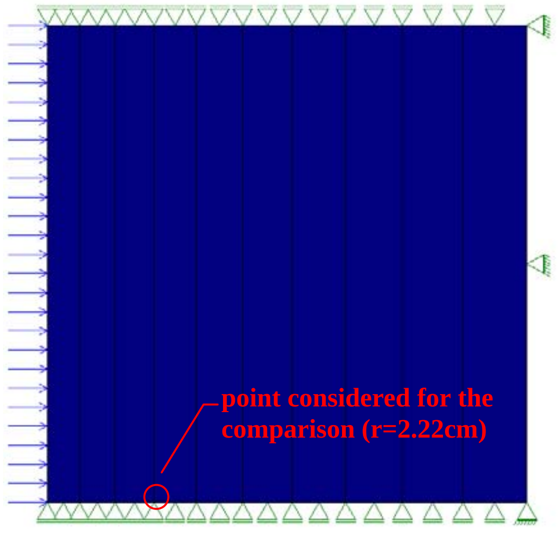
In this section, we validate the object oriented finite element code applied to viscoelastic materials. For this validation, a thick-walled viscoelastic cylinder has been selected. This cylinder is encased in a shell of infinite stiffness and subjected to internal pressure p. The geometry represents a solid propellant rocket motor (Zocher, 1997).

The thick-walled cylinder has an internal (a) and external (b) radius of 2cm and 3cm, respectively. The geometric model, mesh and boundary conditions are illustrated in Fig. 2. We choose to model only ¼ of the entire geometry because of symmetry about its axis. Equation 8 gives the applied load similar to a creep test:

$$p = p_0 H(t) \tag{8}$$

where:
p: applied pressure, p_0: initial pressure of 1kPa and $H(t)$: unit step function.





(a) geometric model (after Zocher, 1997) (b) mesh and boundary conditions

Figure 2 – Geometric model, mesh and boundary conditions of the thick-walled cylinder

An axisymmetric analysis was performed with the isotropic linear viscoelastic property set presented in Tab. 1. This set of properties describes a hypothetical material system, which was designed for this validation.

Table 1. Viscoelastic material properties for the thick-walled pressure vessel

i	E_i (1/kPa)	ρ_i
∞	0.1E+03	-
1	0.4E+03	1.00E+00

Zocher et al. (1997) employed the elastic solution along with the viscoelastic correspondence principle obtained the following analytical solution:

$$u_r(r,t) = \frac{p_0 a^2 b(1+\upsilon)(1-2\upsilon)}{a^2+(1-2\upsilon)b^2}\left(\frac{b}{r}-\frac{r}{b}\right)D(t) \tag{9}$$

where:

u_r: radial displacement.

The employed analytical solution is compared with the numerical results from FE numerical analysis. In order to check the convergence solution, the time was discretized in two values. Figure 3 compares and summarizes the displacement in function of time at a point located in a radius of 2.22 cm (see Fig. 2. Finite element predictions for the viscoelastic cylinder compare well with their corresponding analytic solutions. The convergence was also reached well for a *dt* equal 0.2 seconds.

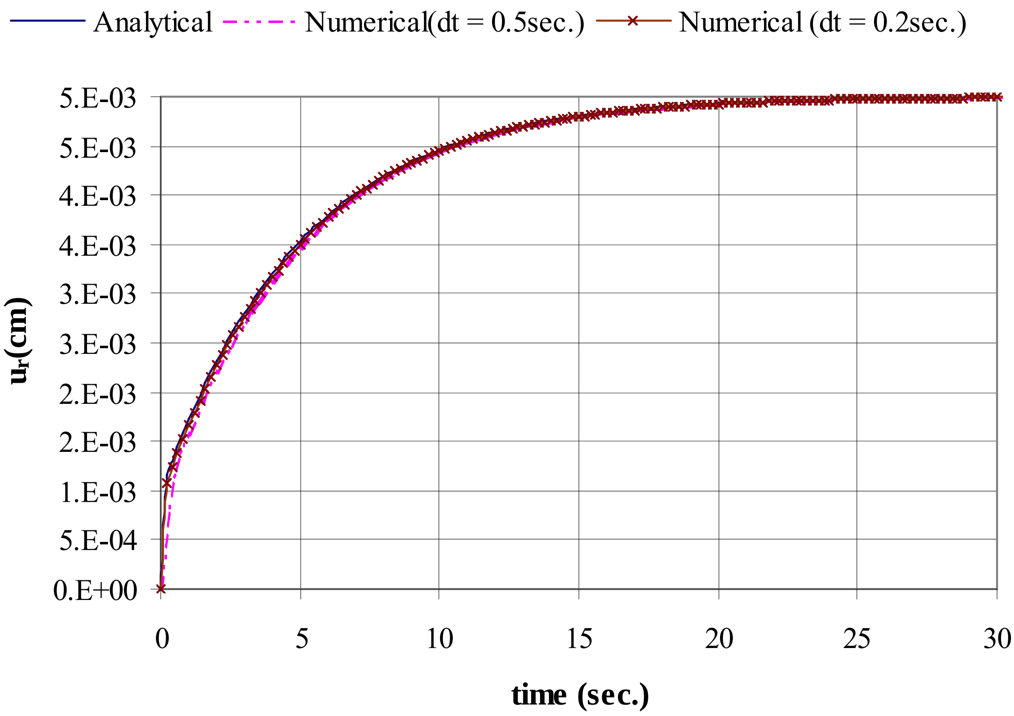


Figure 3 – Comparison between numerical and analytical solutions
for the thick-walled cylinder

3.2 Pavement analyses (geometry, mesh, boundary conditions and material properties)

A static finite element analysis is performed to study stresses and strains in an axisymmetric pavement section. The section is composed by three layers over a granular subgrade (existing ground). The first layer is an asphalt concrete surface of 5 cm. The second and third layers are a granular base and subbase, with 15 cm and 20 cm, respectively.

Typical FE geometric model, mesh and boundary conditions were used in the analyses and are shown in Fig. 4. The axisymmetric model has a radius of 216 cm and a total depth of 431 cm. This is based in the study of Duncan et al. (1968) which recommends a radial limit of 20 times the load radius and a thickness of the subgrade layer of 40 times the load radius. The model includes 1900 8-node quadratic elements with 2x2 gauss points.

A load of 0.56 MPa, single wheel load, is applied on the surface in a circular area with a radius of 10.8 cm to simulate the load of an 8.2tf-standard axle. In order to investigate the effect of the viscoelastic behavior under vehicle speed over the asphaltic surface layer, 4 sine half-wave pulses of 0.01sec, 0.015sec, 0.1sec equivalent to speeds of 100 km/h, 60 km/h and 10 km/h were calculated by Eq. (4). A load of a 1 second-duration in the form of a step function (Eq. 8) is also applied to simulate a static load. At graphical figures the load durations is reported as t_c (time of cycle).

Two types of material are used in modeling the layered system. Isotropic elastic constitutive models are used for the granular materials (base, subbase and subgrade). The elastic material properties are shown in Tab. 2 and they are typical values for this type of granular material used in Brazil (Soares et al., 2000) and also used in Souza and Soares (2003).

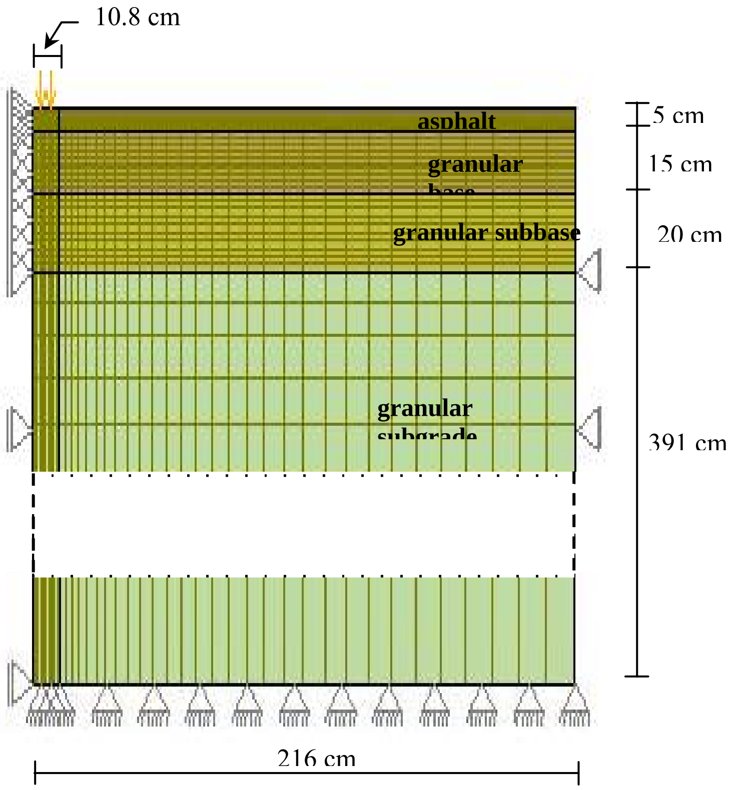


Figure 4 – Geometric model, mesh and boundary conditions of the pavement structure

Table 2. Elastic material properties for granular materials

Layer	E (MPa)	υ
Base	300.00	0.30
Subbase	200.00	0.30
Subgrade	100.00	0.30

The asphalt surface layer is simulated as a viscoelastic material. The Prony series coefficients were extracted from Lee (1996) for the creep compliance as shown in Tab. 3. This asphalt mixture is a HMA fabricated under SHRP (1994) specifications. A constant Poisson ratio of 0.35 is assumed.

Table 3. Viscoelastic material properties for asphalt concrete layer (after Lee, 1996)

i	D_i (1/MPa)	τ_i
∞	6.89E-05	-
1	9.61E-06	5.00E-06
2	1.63E-05	5.00E-05
3	4.92E-05	5.00E-04
4	1.12E-04	5.00E-03
5	2.26E-04	5.00E-02
6	5.75E-04	5.00E-01
7	3.53E-03	5.00E-00
8	8.91E-03	5.00E-01
9	6.24E-02	5.00E-02
10	3.85E-01	5.00E-03
11	3.92E-01	5.00E-04

In order to compare the structural response of the surface layer when considering of linear viscoelastic and elastic behavior, it is necessary the determination of *E* of the mixture. As the values presented in Tab. 3 were extracted from the literature, it is not possible to perform laboratorial tests for the mixture to determine its modulus of elasticity. However, with the viscoelastic properties we can numerically simulate a uniaxial test to determine the required elastic parameter as done in Souza and Soares (2003) based in the observations of Christensen (1982) and Allen and Haisler (1985).

E of asphalt mixtures can be calculated from the RM test. In Brazil the determination of RM is tipically one through the diametrical compression test, but the RM can also be obtained from a uniaxial compression test (Huang, 1993). This test consists in a sine half-wave of 0.1sec of load application (a maximum of 0.1MPa) and 0.9sec. of rest. The deformation in this period (1 sec) is recoverable and constant during each cycle, although the total deformation increases due to non-recoverable deformations. This test is better detailed in ASTM (1982) and DNER (1994). The RM can be obtained from Eq. 10:

$$RM_{HMA} = \frac{\sigma_0}{\varepsilon_{\max} - \varepsilon_{end}} \tag{10}$$

where: σ_0: maximum applied stress; $\varepsilon_{\max}$: maximum strain obtained, and ε_{end}: strain at the end of the load cycle.

Figure 5 shows the strain results for the determination of RM for a pulse of 0.1sec. Using Eq. (8), the MR determined for the HMA used herein was 2555.16MPa. Souza and Soares (2003) observed that RM does not represent just an elastic parameter for asphalt mixtures. This is due to viscoelastic deformations counted as elastic deformations during the load pulse. In other words, the authors showed that the elastic contribution in the total deformation is small when compared to the viscoelastic contribution.

In order to verify this observation the same uniaxial compression test is simulated, but with a pulse of 0.01sec by 0.09sec of rest (10% of the standard pulse of 0.1sec.). The result is also showed in Fig. 5. It noted that the strains decrease almost by 2 times for the 0.01sec pulse, consequently, the RM value increases about 89% (4838.80MPa). With this more instantaneous application of stress, just the spring in the mechanical analog is mobilized (E_0 in Fig. 1). The springs associated in parallel with the dashpots, have less time to be mobilized. Their deformations are time dependent and they are related to the dashpot's relaxation. The mathematical proof of this finding is demonstrated in Allen (1997).

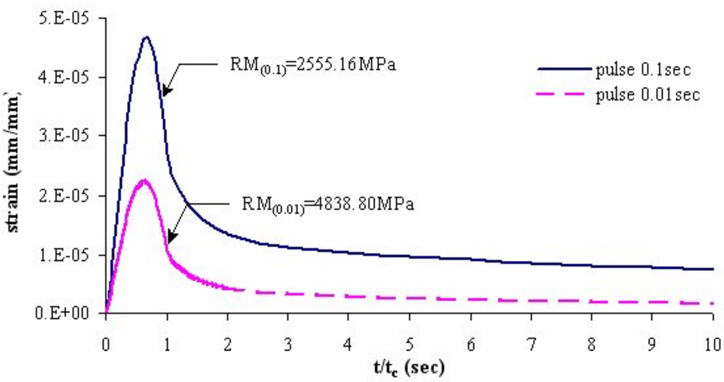


Figure 5 – Strains obtained in numerical simulation of the uniaxial RM test

Therefore, the are two very different values for the RM depending on load time duration. In the next sections the RM determined by the 0.1sec. pulse (2555.16MPa) is used, which is recommended by Brazilian (DNER, 1994) and international specifications (ASTM, 1982).The above mentioned analysis is just to illustrate that the RM calculated by the former pulse is not a purely elastic parameter.

4. RESULTS

Figure 6 shows the calculated displacements at the top of the asphalt layer for the node placed at the symmetry axis. It is noted that, when viscoelastic behavior is considered, increasing the load duration more the values of displacements. It is can be a reason for the larger deflections found in urban pavements where vehicles flow in lower speeds. In the case of static load we observe the maximum values of displacements. When compared the two material models (elastic and viscoelastic), the elastic response is similar to the vicoelastic one at 0.1sec pulse. This is expected because the *E* was obtained by the 0.1sec. pulse in the RM test. It is a confirmation of the study of Souza and Soares (2003) who observed that the RM is not only an elastic parameter. In the static cases, the considerations of elasticity for the asphalt layer tends to turn more stiffness the structure as realized from the less values of displacements in the elastic approach.

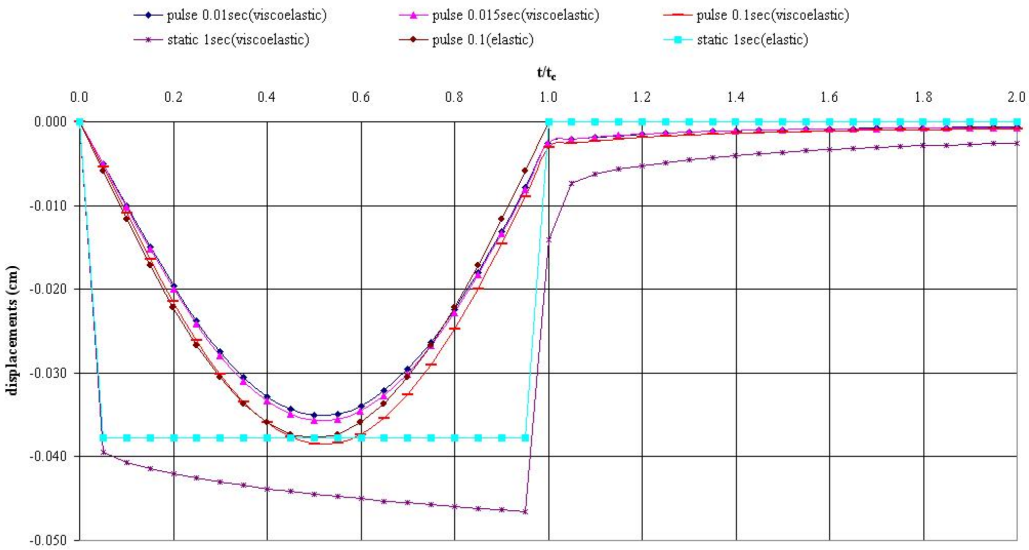


Figure 6 – Displacements at the top of asphalt layer

Figure 7 shows a quite different behavior in the induced horizontal stresses (σ_{xx}) of asphalt layer where the stress value was observed at the closest gauss point both to symmetry axis and bottom of layer. Longer duration pulses induce fewer values of horizontal stresses. This fact has the same explanation of the displacements responses: the more instantaneous (shorter) pulses induced the elastic component of the generalized Voigt's model (E_0 in Fig. 1) and the mechanical response tends to be stiffer than on found in longer pulses duration. In those cases, the dashpots in parallel with springs also work in a relaxation way. Almost all cases presents tensile stresses are induced during the pulse duration. It is just not observed in the viscoelastic static simulation. In the end of load we note that, excepting the elastic consideration where a fully-instantaneous recovery is reached, compressive stresses are induced at the bottom. This is explained by the elastic behavior of the sublayres which immediately returns when no more loads are applied. This is important because maybe explain the more useful life of the asphalt mixtures on-service when compared with the laboratory tests predictions (Kim et al. 1990; Little et al., 2002; Souza and Soares, 2003).

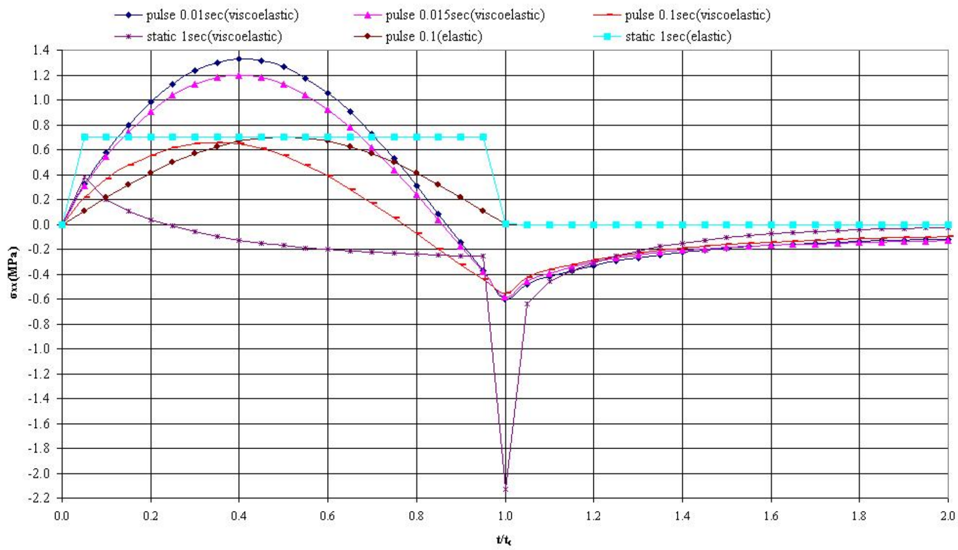


Figure 7 – Tensile stress at the bottom of asphalt layer

Results for predicted vertical stresses (σ_{yy}) at the top of granular subgrade are shown in Fig. 8. The load duration affects the vertical stress magnitude in a direct way similar to displacement response as shown in Fig. 6. Larger value of stress is found in longer pulses where the viscoelastic static case has the maximum absolute stress value.

In the pavement design just the maximum absolute values of the above cited structural parameters are need. Table 4 presents these values in order to quantify the influence of the viscoelastic behavior. Respect to the displacements at the asphalt layer the difference between the two considerations (elastic and viscoelastic) are more relevant when longer pulse duration is applied. As the RM was determined at the 0.1sec.-pulse there is no difference between the viscoelastic and elastic response for this time duration, but for the static load, this difference reaches 18.76% in which the elastic consideration is not safety. Decreasing differences are observed when pulse duration increases (until 0.1sec. pulse). For these cases the viscoelastic calculated displacements are shorter than elastic ones. Similar differences and behavior is observed in vertical stresses (σ_{yy}).

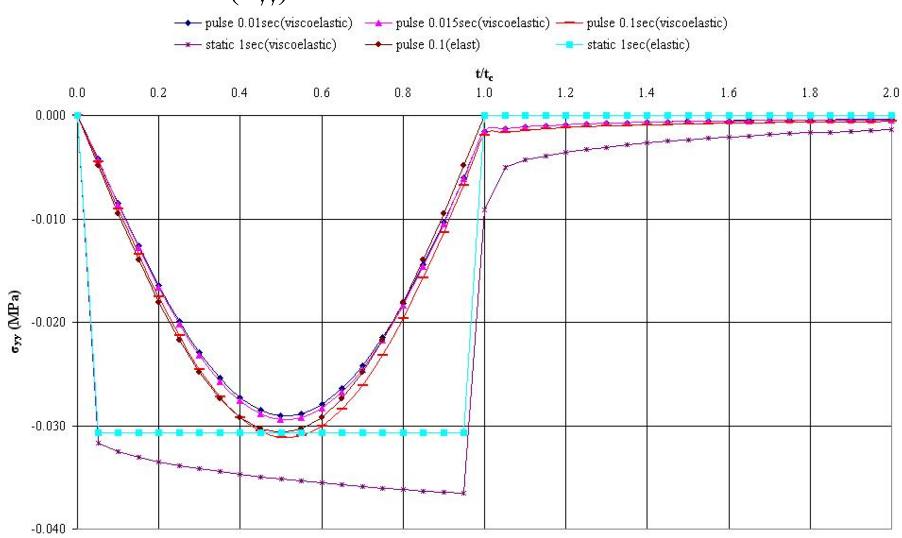


Figure 8 – Compressive stress at the top of subgrade layer

In the analysis of horizontal stresses (σ_{yy}) induced at the bottom of asphalt layer, careful considerations are need. As commented for vicoelastic considerations, shorter pulses causes larger horizontal stresses and the static consideration presents the lowest value for that stress. The most important observation is that the differences between the responses along the pulse duration values are more significant for this parameter (σ_{xx}).Consequently, this response is more sensitive to the applied load. When compared the two material models (elastic and viscoelastic), the differences are also significant reaching about 84%.

Table 4. Summary results for structural parameters for pavement design

Load	Displ. (cm)			σ_{xx} (MPa)			σ_{yy} (MPa)		
	viscoel.	elastic	differ. (%)	viscoel.	elastic	differ (%)	viscoel.	elastic	differ (%)
pulse 0.01sec	0.035	0.038	7.90	1.327	0.702	47.11	0.029	0.031	5.83
pulse 0.015sec	0.036	0.038	6.10	1.198	0.702	41.41	0.029	0.031	4.53
pulse 0.10sec	0.038	0.038	0.00	0.657	0.702	6.74	0.031	0.031	1.30
static 1sec	0.047	0.038	18.76	0.382	0.702	83.55	0.037	0.031	16.00

5. SUMMARY AND CONCLUSIONS

The present study shows the importance of consideration about viscoelastic behavior of asphalt materials in flexible pavements. A FE code was implemented and validated in C++ language and using OOM with elastic and viscoelastic as constitutive models.

This work provides insight information about three parameters used in pavement design: (i) displacement at the top of asphalt surface layer; (ii) tensile stress at the bottom of asphalt surface layer (σ_{xx}) and (iii) compression stress (σ_{yy}) at the top of granular subgrade layer. Numerical analysis shows the difference in these design parameters when the asphalt layer is considered either elastic or viscoelastic. The applied loads are pulse loads, which duration is related with a vehicle speed, and static load. Under the viscoelastic consideration can be observed that increasing the load duration the more displacement and vertical stress (σ_{yy}) values. This behavior shows how the time and load rate dependency of one viscoelastic layer affects the whole layered structure. This is also one reason for the larger rutting found in urban areas due to the lower vehicles speed.

It is important observe that, when viscoelastic behavior is considered, a careful study of the pulse duration should be performed. This is because the tensile stresses (σ_{xx}) have different behavior of displacements and compressive stresses (σ_{yy}) when increase (or decrease) the pulse duration. In this way, the definition of pulse duration for each (or all) structural parameter observed should be thought in terms of either a fixed vehicle speed or the worst "idealized" situation for design purposes.

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