A Distributed Approach for Antenna Subset Selection in MIMO Systems

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***Abstract*—In this paper a novel antenna subset selection algorithm is proposed using a distributed approach. It is assumed that each base station in a group of base stations is linked to an associated terminal as a receiver-transmitter pair. These receiver-transmitter pairs reuse channel resources, such that each mobile terminal represents a source of other-cell interference (also referred to as multi-user interference or MUI) for other mobile terminals in neighboring cells that are reusing all or some of the same channel resources. Accordingly, the base stations implement a gaming-based algorithm to mitigate MUI for the multiple-input-multiple-output (MIMO) uplink signals received from their associated mobile terminals. Simulation results show that the proposed algorithm has a good performance in terms of average error probability consisting of a solution concept based on Nash equilibrium (NE) points.**

I. Introduction

In a scenario where higher and higher spectral efficiency is necessary, sharing of resources is mandatory. This issue requires flexible networks and also yields a competitive nature in modern communication systems.

Multiple transmit and receive antennas (for MIMO transmit/receive processing) can be used to mitigate multi-user interference (MUI) if they are include some intelligent transmission technique. For instance, the use of directional antennas and antenna arrays has long been recognized as an effective technique to reduce MUI [1]. If multiple antennas are also employed to perform spatial multiplexing (SM), where data are transmitted over multiple transmit antennas [2], the spectral efficiency can be further increased.

Different criteria have been used for the antenna subset selection such as maximizing the channel capacity [3], maximizing the post-processing signal-to-noise ratio (SNR) [4] and maximizing the minimum singular value (MSV) of the channel matrix [4]. Also, it is possible to perform the antenna subset selection through centralized optimization by exhaustive search over all possible antenna combinations. However, an exhaustive search approach might be not feasible in practical systems due to the high computational complexity and excessive signaling load. Game theory has also been adopted to solve many problems in communication systems by modeling such systems in a distributed way [5], [6], [7], [8].

As for multi-cell and multi-user communication, multi-cell optimization for diversity multiple-input-single-output (MISO) schemes and interference mitigation has been studied in [9], where a game-theoretic framework is used for a 2-cell scenario in downlink communication and each base-station (BS) aims at the maximization of its error probability making use of partial channel state information (CSI). In addition to this, [10] has generalized antenna selection algorithms proposed in [11], [12] for interference limited MIMO wireless environments. In that work, the antenna selection criterion is the maximization of the post-processing signal-to-interference-plus-noise ratio (SINR) at each BS through a non-iterative algorithm. However, [9] does not consider MIMO configurations and [10] does not use an iterative algorithm in order to mitigate the inherent interference. The works [13], [14], [6] have applied game theory in multi-user MIMO systems. Those works contribute with a general game-theoretic framework using the iterative waterfilling (IWF) algorithm [15] to find out optimal precoding matrices for SM systems. Also, they derive sufficient condition (e.g., convex precoder set) ensuring existence and uniqueness of the Nash equilibrium.

Based on those reference works, this paper aims at providing a game interpretation for a SM system applying antenna subset selection in an environment with MUI. This work particularizes the general framework in [13], [14], [6] for a simpler one which performs antenna subset selection instead of optimal precoding, but the presentation of proof of convergence for a NE is out of the scope of the present work.

The following notation is used throughout this paper. Uppercase and lowercase boldface denote matrices and vectors, respectively. The operators $\mathbb{E}\{\,\cdot\,\}$, $\|\cdot\|$, $\mathrm{D}[\,\cdot\,]$, $|\cdot|$, $(\cdot)^H$ and $\mathrm{tr}(\cdot)$ stand for expectation, norm operator, decision operator, modulus, hermitian and trace operator, respectively.

The paper is organized as follows. Section II describes the system model considered in this work and formulates the optimization problem. In Section III, we introduce our criterion and algorithm by solving the optimization problem via game theory. In Section IV, we report some numerical results obtained which highlight the outcomes of our algorithm. Finally, we state our conclusions in Section V.

II. System Model

Consider a multiuser scenario with K users spread over Q cells. The reuse factor is equal to 1 and there is no intracell

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interference. On the other hand, there are co-channel transmit-receiver pairs (links) in uplink communication that share time and bandwidth resources causing intercell interference. Therefore, for a given set of resources, there are at most Q neighboring links, each user equipment (UE) in a cell, that interfere with each other. Thus, considering the worst case, the set of neighboring links is defined as $\Gamma = \{1, \ldots, Q\}$. In addition to this, each BS may be connected to a base-station controller (BSC) through, for example, a high-speed wired link in order to exchange information. Fig. 1 illustrates a 2-cell scenario where two items of UE share resources (the remaining $K - 2$ users are omitted). For convenience, each item of UE is simply referred to as a UE.

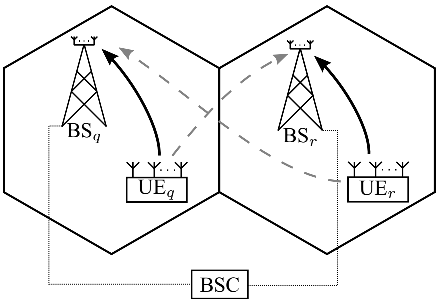

Fig. 1. Example of a general 2-cell scenario.

The UE_q is the q-th source that transmits precoded and spatially multiplexed symbol vectors $\mathbf{x}_q$ to the q-th BS (BS_q). The symbol vectors $\mathbf{x}_q$ are defined as

$$\mathbf{x}_q = \sqrt{\frac{1}{N}} \mathbf{F}_q \mathbf{s}_q, \quad \forall q \in \Gamma, \tag{1}$$

where $\mathbf{F}_q$ is the $M_T \times N$ precoding matrix and $\mathbf{s}_q$ is the $N \times 1$ vector of SM symbols. The BS_q, as the q-th destination, also receives interfering signals from the other $Q - 1$ links. Also, one may assume M_T, N and M_R as being the number of available transmit antennas, the number of radio frequency (RF) chains and the number of receive antennas, respectively.

The sampled symbol vector received by the q-th BS is

$$\mathbf{y}_q = \mathbf{H}_{qq} \mathbf{x}_q + \sum_{r=1, r \neq q}^{Q} \sqrt{g_{rq}} \mathbf{H}_{rq} \mathbf{x}_r + \mathbf{n}_q, \tag{2}$$

where $\mathbf{H}_{qq}$ is the channel matrix between source q and destination q and $\mathbf{n}_q$ is the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise vector with covariance matrix $N_o \mathbf{I}$. On the right-hand side of Eq. (2), the second term refers to the MUI caused by the other links and received by the q-th BS. The fading between each transmit and receive antenna is assumed to be independent, modeled by ZMCSCG random variables and *quasi*-static over a data block of L symbols. Also, it is assumed that each BS knows the CSI for its associated UE perfectly. Further, each BS knows the CSI for the other, interfering UEs. The constant g_{rq} is a gain that depends on the path loss of each interfering signal. Eq. (2) shows a competitive nature of the process of the received signal and makes the use of a game formulation appealing.

For each UE, the average transmit power is constant and given by

$$\mathbb{E}\left\{\|\mathbf{x}_q\|^2\right\} = \frac{1}{N} \mathrm{tr}\left(\mathbf{F}_q \mathbf{F}_q^H\right) = P_q, \quad \forall q \in \Gamma, \tag{3}$$

where P_q is the average transmitted power in units of energy per signaling period. Also, the symbols are assumed to be uncorrelated, which means that $\mathbb{E}\left\{\mathbf{s}_q \mathbf{s}_q^H\right\} = \mathbf{I}$.

At each receiver, the MUI is treated as additive noise. This assumption is due to the fact that interference cancellation algorithms need some information (e.g., CSI) from interfering users [16] increasing the system signaling load. Hence, the estimated symbol vector at the q-th BS is defined as $\hat{\mathbf{s}}_q = \mathrm{D}\left[\mathbf{G}_q^H \mathbf{y}_q\right]$, where $\mathbf{G}_q$ represents the minimum mean-square error (MMSE) stage [17], [13] and it is defined as

$$\mathbf{G}_q = \mathbf{R}_{-q}^{-1} \mathbf{H}_{qq} \mathbf{F}_q \left(\mathbf{I} + \mathbf{F}_q^H \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1} \mathbf{H}_{qq} \mathbf{F}_q\right)^{-1}. \tag{4}$$

The term $\mathbf{R}_{-q}$ corresponds to the interference-plus-noise covariance matrix estimated by the q-th BS. Also in [17], [13], this matrix $\mathbf{R}_{-q}$ is defined as

$$\mathbf{R}_{-q} \triangleq N_o \mathbf{I} + \sum_{r=1, r \neq q}^{Q} |g_r| \mathbf{H}_{rq} \mathbf{F}_r \mathbf{F}_r^H \mathbf{H}_{rq}^H$$

which is clearly a function of the interfering signals. The subscript $-q$ denotes all the players belonging to Γ except the q-th player.

Before transmitting, each UE selects a precoding matrix $\mathbf{F}$, which is related to an antenna subset, Generally, for a given UE, the selection of $\mathbf{F}$ is based on some information fed back by the BS with which the UE is associated, as illustrated in Fig. 2.

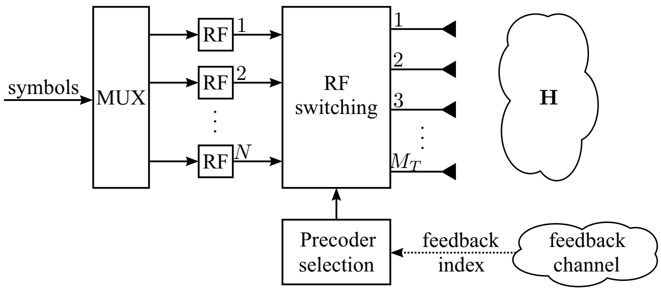

Fig. 2. Block diagram of a user equipment.

Consider a codebook $\mathcal{W}$ as being the set of all precoding matrices available for every entity in the system (e.g., for all UEs). For purposes of antenna subset selection, one may define each element of $\mathcal{W}$ as a $M_T \times N$ submatrix of an identity matrix $\mathbf{I}$. That is, the unique non-null entry of each column of this submatrix selects a transmit antenna. In order to index the elements of $\mathcal{W}$, assume an index set defined as $\mathcal{I} \triangleq \{1, 2, \ldots, \binom{M_T}{N}\}$. Thus, a bijective function $f : \mathcal{I} \leftrightarrow \mathcal{W}$ maps the elements of $\mathcal{I}$ onto the elements of $\mathcal{W}$ properly. For example, for $M_T = 3$ and $N = 2$:

$$\mathcal{W} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \quad \mathcal{I} = \{1, 2, 3\}$$

$$f(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad f(2) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \quad f(3) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

For the sake of simplicity, it may be assumed that every receiver-transmitter pair has the same configuration, i.e., the same number of RF chains, and transmit and receiver antennas. Therefore, each receiver-transmitter pair works with the same codebook $\mathcal{W}$.

In the following, we will introduce the formulation of our antenna subset selection game as well as the selection criteria adopted in this work.

III. Antenna Subset Selection Game

We propose a precoder matrix selection game which employs a game theory tool to solve the precoding selection problem, based on exploiting its interesting feature of solving optimization problems in a non-centralized way. According to the game, each BS uses the known (or indirectly estimated) precoder matrix selections made by the other BSs for their respective UEs, to estimate the covariance of interference and noise at the BS for its UE's uplink signal. Each BS then uses that covariance estimate to determine the precoder matrix selection that optimizes in some sense the reception of its UE's uplink signal.

From the system model in Eq. (2), the SINR in the k-th data stream after the MMSE stage at the q-th BS is given by [13] as

$$\text{SINR}_{k,q} = \frac{1}{\left[\left(\mathbf{I} + \mathbf{F}_q^H \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1} \mathbf{H}_{qq} \mathbf{F}_q\right)^{-1}\right]_{k,k}} - 1. \tag{5}$$

From Eq. (5), one can see that there exist conflict of interests among the players, since $\mathbf{R}_{-q}$ is a function of the precoding matrices chosen by the interfering users. Thus, $\mathbf{R}_{-q}$ is the information that the q-th player has to realize at each game iteration. Hence, one may advantageously define the utility function of the q-th player as follows below:

$$u_q(\mathbf{F}_q, \mathbf{F}_{-q}) = \min_k \text{SINR}_{k,q}, \quad \forall k \in \mathcal{N},\ \forall q \in \Gamma, \tag{6}$$

where $\mathcal{N} \triangleq \{1, \ldots, N\}$ is the set of symbol streams. The motivation for maximizing the minimum SINR comes from the intuition that the performance of the receiver should improve as the smallest value of the SINR increases [4]. Here, the "smallest" SINR value is the minimum per-stream SINR, for the multi-stream MIMO uplink between a given one of the BS's playing the game, and its associated UE. We are focusing on the configuration of the selection of the precoder for a block of resources and not for the full bandwidth.

A. Game Formulation

We consider each neighboring link as a rational decision-maker, i.e., a player in the game. From the game standpoint, each player contends for the maximization of its own SINR. In practice, each player's strategy is to select one of the precoding matrices in $\mathcal{W}$ after determining or otherwise obtaining the information $\mathbf{R}_{-q}$ in a game iteration.

Let $\mathcal{G}_1$ be the non-cooperative and nonzero-sum game. Stated in mathematical terms, $\mathcal{G}_1$ has the following structure:

$$(\mathcal{G}_1): \quad \begin{cases} \underset{\mathbf{F}_q}{\text{maximize}} & u_q(\mathbf{F}_q, \mathbf{F}_{-q}) \\ \text{subject to} & \mathbf{F}_q \in \mathcal{W}, \end{cases} \quad \forall q \in \Gamma \tag{7}$$

where $\mathcal{W}$ is the codebook known by all the players.

We define the solution of the game $\mathcal{G}_1$ as being a NE point. This kind of equilibrium is established if each player has chosen an action and no one can benefit by changing its action unilaterally while the other ones keep theirs unmodified [18]. Therefore, an action tuple $\{\mathbf{F}_q^\star, \mathbf{F}_{-q}^\star\}$ is a NE if

$$u_q(\mathbf{F}_q^\star, \mathbf{F}_{-q}^\star) \geq u_q(\mathbf{F}_q, \mathbf{F}_{-q}^\star), \quad \forall \mathbf{F}_q \in \mathcal{W},\ \forall q \in \Gamma. \tag{8}$$

The superscript $\star$ denotes that the underlying precoder leads to a NE. The structure above is a convenient form for representing a NE [18]. Thus far, sufficient conditions for the existence of a NE has not been identified. From [18], [6], [19], some standard results from fixed-point theory and contraction maps are used to state this conditions. One requires a nonempty, convex and compact codebook $\mathcal{W}$ to guarantee the existence of at least one NE. However, the codebook design adopted in a real-world communication system does not necessarily satisfy such requirements. Hence, another antenna selection algorithm, the maximum minimum singular value (MMSV) algorithm proposed in [4], is made available is cases where equilibrium is not reached (e.g., within an allowed number of game iterations).

B. GRASS Algorithm

The process of reaching an equilibrium point is an important issue and it is usually described by a distributive algorithm. Thus, this work defines a (distributed) algorithm for antenna subset selection referred to as the Game-theoRetic Antenna Subset Selection (GRASS) algorithm. The GRASS algorithm is performed at each BS with no coordination among the UEs.

TABLE I
Summary of the GRASS algorithm.

Initial:	estimation step at each BS
Iterations:	BSs exchange information BSs tries to reach the NE point $\mathcal{G}_1$ is played until all users converge
End:	BSs sends the index $i^\star$ back to their users

The algorithm may be summarized (Table I) as: (1) Performing an initial step of channel estimation at BS; and, (2) in each of a bounded number of iterations, the BS (via the BSC) exchange information about the precoder matrix selection made for their respective UEs, with each BS trying to reach the NE point, and with game play continuing until all BSs converge (or until a limit number of iterations is reached).

A loop *counter* controls the game iterations and it is upper-bounded by the a constant defined as $\lambda = \left[\binom{M_T}{N}\right]^Q$. In fact, the value of λ equals the number of all possible action tuples. Therefore, the MMSV algorithm is triggered if and only if no point of equilibrium is found in λ iterations. The finalized precoding matrix selection (represented by its index $i^\star$) arrived at by each BS is sent to the UE associated with that BS.

IV. Simulation Results

We evaluate the bit error rate (BER) averaged over at least 10^6 channel realizations via Monte Carlo simulations. A binary phase shift keying (BPSK) modulation is used as well as a data block length $L = 102$ symbols in each transmission setup. Also, channel realizations are independent identically distributed (i.i.d) from block to block. Here, the structure $(M_T, N) \times M_R$ means that the system selects N transmit antennas out of M_T and receives the transmitted signal with M_R antennas.

The analysis considers a scenario with only two users, i.e., 2 UEs, with varying signal-to-interference ratio (SIR) values observed at each BS. The UEs are positioned such that each BS observes the same SIR. Thus, it is enough to illustrate only the average bit error rate (BER) curves. The algorithms used as reference cases are the MMSV, and the exhaustive search, which is used as a performance bound. In view of showing the diversity gain due to the antenna selection process, some curves of SM MIMO scheme with no antenna selection are shown.

In Fig. 3, the GRASS algorithm has a performance loss compared to the lower bound represented by the (computationally expensive) exhaustive search. It is worth noting that the lower bound curve is drawn from a centralized algorithm that yields an optimal performance, whereas the GRASS algorithm is suboptimal. However, the GRASS algorithm provides for a decentralized approach, which offers significant advantages when used in a wireless communication. Besides that significant advantage, the performance of the GRASS algorithm is significantly close to the optimal. For a BER target equal to 10^{-2}, the penalty is approximately 1.3 dB.

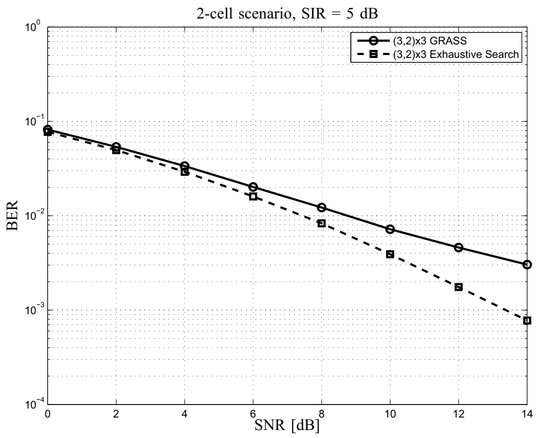

Fig. 3. Bit Error Rate - $(3, 2) \times 3$ System.

In Fig. 4, one sees that the proposed game approach—the use of GRASS—achieves a lower BER floor compared to MMSV. This performance advantage arises because the GRASS algorithm inherently mitigates MUI.

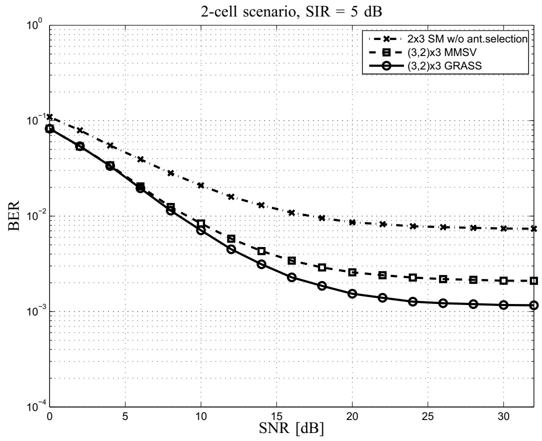

Fig. 4. Bit Error Rate - $(3, 2) \times 3$ System.

On that point, as MUI decreases, the conflict aspect of the game is diminished. That is, there is no significant mutual interference between the links in high SIR regimes. Therefore, the game solution approaches the reference single user case in [4]. This behavior can be seen in Figs. 5 and 6. In the former, the obtained performance gain is lower compared to Fig. 4, whereas in the latter the GRASS curve has almost no gain compared to the MMSV curve.

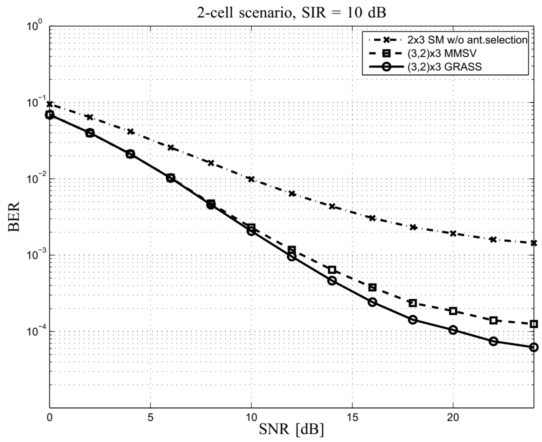

Fig. 5. Bit Error Rate - $(3, 2) \times 3$ System.

In Fig. 7, the NE probability decreases as the mutual MUI increases and becomes dominant compared to the noise factor in the denominator of Eq. (5). Consequently, the number of game iterations increases since the lack of NE implies the use of the fall-back algorithm MMSV.

V. Conclusions and Perspectives

Spatial multiplexing schemes are broadly used in modern communication systems since they provide capacity gain

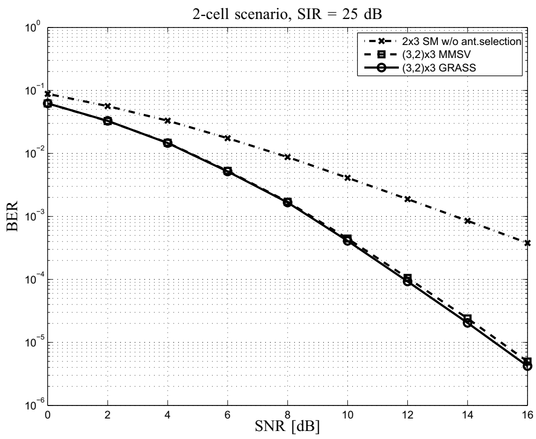

Fig. 6. Bit Error Rate - $(3, 2) \times 3$ System.

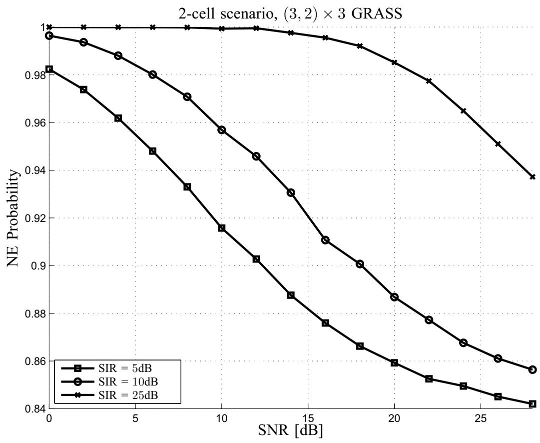

Fig. 7. Probability of the existence of a Nash Equilibrium.

through the employment of multiple antennas. In such systems, the cost of using several RF chains is high. Therefore, antenna subset selection algorithms emerge as a feasible way to decrease this cost.

In this work, a criterion for antenna selection was proposed based on a game theoretical approach that considers a competitive multi-user environment. The proposed algorithm maximizes the minimum per-stream SINR and Monte Carlo simulations show that there is a significant gain compared to the criteria that does not consider MUI in terms of BER. Such gain increases when MUI increases until certain level in which the interference can be mitigated by the MMSE receiver filter. The GRASS algorithm can therefore be useful to improve the performance of MIMO schemes providing a number of significant performance and implementation advantages, for many real-world operating scenarions.

Future work includes the analysis of the system performance with the use of new codebook designs, for the purpose of ensuring the existence of NE, and extension of our criterion to Hybrid MIMO Transceivers which take advantage of the fundamental trade-off between diversity and multiplexing gains [20], [21].

ACKNOWLEDGMENT

This work was partially supported by the Research and Development Centre, Ericsson Telecomunicações S.A., Brazil, under EDB/UFC.23 Technical Cooperation Contract.

REFERENCES

[1] Paulraj, A., Nabar, R., and Gore, D., *Introduction to Space-Time Wireless Communications*. Cambridge, UK: Cambridge University Press, 2003.

[2] Telatar, Emre, "Capacity of Multi-antenna Gaussian Channels," *European Transactions on Telecommunications*, vol. 10, pp. 585–595, 1999. [Online]. Available: http://citeseer.ist.psu.edu/346880.html

[3] Nabar, R. U., Gore, D. A., and Paulraj, A., "Optimal selection and use of transmit antennas in wireless systems," in *International Conference on Telecommunications (ICT)*, Acapulco, Mexico, 2000.

[4] Heath, Jr., R. W., Sandhu, S., and Paulraj, A., "Antenna selection for spatial multiplexing systems with linear receivers," *IEEE Communications Letters*, vol. 5, no. 4, pp. 142–144, April 2001.

[5] MacKenzie, A. B. and DaSilva, L. A., *Game Theory for Wireless Engineers*. USA: Morgan & Claypool, 2006.

[6] Scutari, G., Palomar, D. P., and Barbarossa, S., "Competitive design of multiuser MIMO systems based on game theory: A unified view," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 7, pp. 1089–1103, September 2008.

[7] Menon, R., Mackenzie, A., Hicks, J., Buehrer, R., and Reed, J., "A game-theoretic framework for interference avoidance," *IEEE Transactions on Communications*, vol. 57, no. 4, pp. 1087–1098, April 2009.

[8] Larsson, E., Jorswieck, E., Lindblom, J., and Mochaourab, R., "Game Theory and the Flat-Fading Gaussian Interference Channel," *IEEE Signal Processing Magazine*, vol. 26, no. 5, pp. 18–27, Feb. 2009.

[9] Oteri, O. and Paulraj, A., "Multicell optimization for diversity and interference mitigation," *IEEE Transactions on Signal Processing*, vol. 56, no. 5, pp. 2050–2061, 2008.

[10] Sigdel, S. and Krzymien, W. A., "Interference suppression through adaptive subset antenna transmission in interference limited MIMO wireless environments," in *VTC Fall*, 2006, pp. 1–5.

[11] Gore, D., Heath, Jr., R. W., and Paulraj, A., "Transmit selection in spatial multiplexing," *IEEE Commun. Lett*, vol. 6, pp. 491–493, 2002.

[12] Heath, Jr., R. W. and Paulraj, A., "Antenna selection for spatial multiplexing systems based on minimum error rate," in *Communications, 2001. ICC 2001. IEEE International Conference on*, vol. 7, Helsinki, 2001, pp. 2276–2280.

[13] Scutari, G., Palomar, D. P., and Barbarossa, S., "Optimal Linear Precoding Strategies for Wideband Noncooperative Systems Based on Game Theory - Part I: Nash Equilibria," *IEEE Transactions on Signal Processing*, vol. 56, no. 3, pp. 1230–1249, March 2008.

[14] ——, "Optimal linear precoding strategies for wideband non-cooperative systems based on game theory - part II: Algorithms," *IEEE Transactions on Signal Processing*, vol. 56, no. 3, pp. 1250–1267, March 2008.

[15] ——, "The MIMO iterative waterfilling algorithm," *IEEE Transactions on Signal Processing*, vol. 57, no. 5, pp. 1917–1935, 2009.

[16] Andrews, J. G., Choi, Wan, and Heath, Jr., R. W., "Overcoming interference in spatial multiplexing MIMO cellular networks," *IEEE Wireless Communications*, vol. 14, no. 6, pp. 95–104, December 2007.

[17] Palomar, D. P. and Barbarossa, S., "Designing MIMO communication systems: constellation choice and linear transceiver design," *IEEE Transactions on Signal Processing*, vol. 53, no. 10, pp. 3804–3818, October 2005.

[18] Başar, T. and Olsder, G. J., *Dynamic Noncooperative Game Theory*. Great Britain: Harcourt Brace & Company, 1995.

[19] S. Lasaulce, M. Debbah, and E. Altman, "Methodologies for analyzing equilibria in wireless games," *Signal Processing Magazine, IEEE*, vol. 26, no. 5, pp. 41–52, September 2009.

[20] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.

[21] Freitas Jr., W. C., Cavalcanti, F. R. P., and Lopes, R. R., "Hybrid transceiver schemes for spatial multiplexing and diversity in MIMO systems," in *Journal of Communication and Information Systems (JCIS)*, vol. 20, December 2005, pp. 141–154.