OUTAGE PERFORMANCE OF COOPERATIVE AMPLIFY-AND-FORWARD OFDM SYSTEMS WITH NONLINEAR POWER AMPLIFIERS

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ABSTRACT

Cooperative diversity and orthogonal frequency division multiplexing (OFDM) are two key technologies for future wireless communication systems. One of the main problems of OFDM systems is the high peak-to-average power ratio (PAPR) of the transmitted signals, which may cause the introduction of intercarrier interference due to the presence of nonlinear power amplifiers (PAs). In this paper, a theoretical analysis of the outage probability of an amplify-and-forward (AF) cooperative diversity OFDM system accounting for nonlinear distortions introduced by a nonlinear PA is developed. It is assumed a frequency-selective Rayleigh fading and a downlink transmission, with the base station having a linear PA and the relay having a nonlinear PA. Our analysis shows how the PA parameters affects the outage probability for different SNR levels. The validity of the proposed outage analysis is verified by means of computer simulations.

1. INTRODUCTION

In the last years, cooperative diversity has emerged as a promising technology for wireless communication systems due to its ability to exploit spatial diversity without the need of multiple antennas. Cooperative diversity systems emulate an antenna array in a distributed manner, allowing one or more mobile units to relay the information data from a source node to the destination node [1]. The main objective is to extend coverage, spectral efficiency and capacity while keeping the advantages of multiple-input multiple-output (MIMO) systems. Several cooperative relaying protocols have been proposed in the literature such as the amplify-and-forward (AF), fixed decode-and-forward and selective decode-and-forward [2]. In the present paper, we are interested in the AF protocol due to the fact that it avoids decoding at the relays and, therefore, it is often preferable when complexity or latency issues are of importance.

On the other hand, orthogonal frequency division multiplexing (OFDM) has become the basis of many wireless communication standards, such as IEEE 802.11a, IEEE 802.16, 3GPP LTE and digital video broadcasting return channel terrestrial [3], mainly due to its high spectral efficiency, robustness to multipath fading and low complexity implementation. One of the main problems of OFDM is that the transmitted signals are characterized by a high peak-to-average power ratio (PAPR) [3, 4]. Due to the presence of nonlinear devices such as power amplifiers (PAs), a high PAPR causes the introduction of nonlinear inter-carrier interference (ICI) in the received signals if a high input back-off (IBO) is not used, which can significantly deteriorate the recovery of the information symbols. The IBO is defined as the ratio between the PA saturation power, i.e. the input power corresponding to the maximum output power, and the average PA input power. A high IBO results in a low power efficiency of the PA and a low signal-to-noise ratio (SNR) at the receiver.

In non-cooperative systems, the high PAPR is usually more critical in uplink transmissions, as user terminals have cheaper equipments and stronger power constraints than base stations. Moreover, base stations have more processing resources than user equipments, allowing for the application of techniques to compensate the amplifier nonlinearity at the base station [5, 6, 7]. Indeed, OFDM-based multiple access scheme has been adopted for the downlink in the 3GPP Long Term Evolution (LTE), while single-carrier frequency division multiple access (SC-FDMA) has been adopted for the uplink [3]. In cooperative relay systems, the high PAPR is also an important issue in the downlink, as the nonlinear PA of the relay may introduce significant distortions [8, 9].

In this work, a theoretical analysis of the outage probability of AF cooperative diversity OFDM systems is developed by taking the nonlinear distortions introduced by the relay PA into account. We assume a frequency-selective Rayleigh fading and downlink transmission, with the base station having a linear PA and the relay having a nonlinear memoryless PA. Moreover, we consider a maximal-ratio combining (MRC) receiver to combine the signals received through the direct link and through the relay link. Specifically, the cumulative distribution function (CDF) of the output of the MRC is derived. Our analysis shows that the nonlinear PA decreases the system diversity only for high SNR thresholds. The validity of the proposed outage analysis is verified by means of computer simulations.

To the best of author's knowledge, up to now, only few works have investigated the impact of PA nonlinear distortions in cooperative communication systems and none of them perform outage analysis. In [8], techniques for PA nonlinear distortion cancelation in an OFDM AF cooperative communication system were proposed, considering roughly the same system model as the present work. However, no performance analysis was presented in [8]. In [10], an optimal relay power allocation for AF relay networks with nonlinear PAs was proposed, by considering multiple relays. In [9], a bit error probability expression was developed for an AF cooperative diversity system, assuming that both base station and relay have nonlinear PAs, and that the relay-destination channel is time-invariant. An outage probability analysis of an AF cooperative communication system was developed in [11] taking the impact of the saturation of the mean transmission power into account. However, it does not consider the nonlinear distortions caused by the saturation of the instantaneous amplitude of the transmitted signals, contrary to the present paper. Moreover, in [10, 9, 11], it is assumed that there is no direct path between the source and the destination.

The rest of the paper is organized as follows. Section 2 describes the system model considered in this work. In Section 3, the outage analysis is presented. In Section 4, we evaluate the validity of

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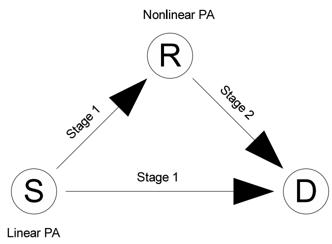

Fig. 1. Cooperative system model.

the proposed outage analysis by means of computer simulations and some conclusions and perspectives are drawn in Section 5.

2. SYSTEM MODEL

A simplified scheme of the cooperative OFDM system used in this work is shown in Fig. 1. An AF cooperative diversity scenario is assumed, where the source (S) communicates with the destination (D) through the direct link during the transmission stage 1, and through the relay (R) link during the transmission stage 2. Time-division multiple-access (TDMA) is employed for orthogonal channel access and all the nodes are equipped with a single antenna operating in half-duplex mode. We assume that the wireless links have frequency-selective Rayleigh fading and that the three nodes are synchronized at the symbol level. Moreover, all the subcarriers of the source have the same transmission power P_s.

As aforementioned, we consider that the source PA is linear while the relay PA is nonlinear. This assumption can be justified by considering a downlink transmission, where the source is a base station with less power constraints and more processing resources than the relay (user terminal).

Assuming that the length of the OFDM cyclic prefix is greater than or equal to the channel delay spread, the discrete-time baseband signal in the frequency-domain $x_n^{(SD)}$ received through the direct link (source-destination) at the n^{th} subcarrier can be written as:

$$x_n^{(SD)} = h_n^{(SD)} \sqrt{P_s}\, s_n + v_n^{(SD)}, \tag{1}$$

for $1 \leq n \leq N$, where N is the number of subcarriers, $h_n^{(SD)}$ is the channel frequency response (CFR) at the n^{th} subcarrier of the source-destination link, s_n is the frequency-domain data symbol at the n^{th} subcarrier and $v_n^{(SD)}$ is the corresponding additive white Gaussian noise (AWGN) component in the frequency-domain. The data symbols s_n ($1 \leq n \leq N$) are assumed to be independent and identically distributed (i.i.d.), with an uniform distribution over a quadrature amplitude modulation (QAM) or phase-shift keying (PSK) alphabet.

The signal received by the relay at the n^{th} subcarrier in the frequency-domain is given by:

$$x_n^{(SR)} = h_n^{(SR)} \sqrt{P_s}\, s_n + v_n^{(SR)} \tag{2}$$

where $h_n^{(SR)}$ is the CFR at the n^{th} subcarrier of the source-relay channel and $v_n^{(SR)}$ is the corresponding noise component. At the relay, the signal $x_n^{(SR)}$ is multiplied by a gain in the following way:

$$q_n^{(R)} = g_n x_n^{(SR)}, \tag{3}$$

for $1 \leq n \leq N$, where g_n is the amplification factor of the n^{th} subcarrier generally given by [1]:

$$g_n = \frac{\sqrt{P_r}}{\sqrt{\left|h_n^{(SR)}\right|^2 P_s + \sigma_v^2}}, \tag{4}$$

where σ_v^2 is the variance of $v_n^{(SR)}$ and P_r is the transmission power of the n^{th} relay subcarrier at the input of the PA.

After computing the inverse discrete Fourier transform (IDFT) of $q_n^{(R)}$ ($1 \leq n \leq N$) and inserting the cyclic prefix, the time domain version of $q_n^{(R)}$ ($1 \leq n \leq N$) is amplified by a PA that is modeled as memoryless function $F(\cdot)$. Thus, we can write:

$$\tilde{u}_{n'}^{(R)} = F\left(\tilde{q}_{n'}^{(R)}\right), \tag{5}$$

for $1 \leq n' \leq N + M_{CP}$, where $\tilde{q}_{n'}^{(R)}$ and $\tilde{u}_{n'}^{(R)}$ are the time-domain signals at the input and output of the PA, respectively, and M_{CP} is the cyclic prefix length. Although we assumed that the function $F(\cdot)$ does not depend on the subcarrier, the analysis developed in the next section can be extended to the case of a PA modeled by a frequency-dependent function. However, for simplicity reasons, it will be considered that the PA function $F(\cdot)$ is frequency-independent.

For a high number N of subcarriers, the signal transmitted by the source in the time-domain $\tilde{s}_{n'}$ can be modeled as a complex Gaussian random variable. Therefore, $\tilde{q}_{n'}^{(R)}$ is also a complex Gaussian random variable. Thus, assuming a rectangular pulse shaping at the transmission and using the extension of Bussgang's theorem to bandpass memoryless nonlinearities with complex Gaussian inputs, the PA output in the time-domain can be expressed as [12]:

$$\tilde{u}_{n'}^{(R)} = K_0^{(R)} \tilde{q}_{n'}^{(R)} + \tilde{d}_{n'}^{(R)}, \tag{6}$$

where $\tilde{d}_{n'}^{(R)}$ is the time-domain nonlinear distortion noise uncorrelated with $\tilde{q}_{n'}^{(R)}$ and $K_0^{(R)}$ is a complex-valued constant given by $K_0^{(R)} = \mathbb{E}[\tilde{u}_{n'}^{(R)} (\tilde{q}_{n'}^{(R)})^*]/P_r$.

In the frequency-domain, eq. (6) becomes:

$$\begin{aligned} u_n^{(R)} &= K_0^{(R)} q_n^{(R)} + d_n^{(R)} \\ &= K_0^{(R)} g_n x_n^{(SR)} + d_n^{(R)}, \end{aligned} \tag{7}$$

for $1 \leq n \leq N$, where $u_n^{(R)}$, $q_n^{(R)}$ and $d_n^{(R)}$ are the discrete Fourier transforms (DFTs) of $\tilde{u}_{n'}^{(R)}$, $\tilde{q}_{n'}^{(R)}$ and $\tilde{d}_{n'}^{(R)}$ ($1 \leq n' \leq N$), respectively, $d_n^{(R)}$ being uncorrelated with $q_n^{(R)}$. The signal $d_n^{(R)}$ is the frequency-domain nonlinear distortion (NLD). For a rectangular waveform, the NLD is a sum of N uncorrelated complex random variables and it can be modeled as a complex Gaussian random variable with variance [12]:

$$\sigma^2_{d^{(R)}} = \sigma^2_{\tilde{d}^{(R)}} = \mathbb{E}\left[\left|u_{n'}^{(R)}\right|^2\right] - \left|K_0^{(R)}\right|^2 P_r \tag{8}$$

where $\sigma^2_{d^{(R)}}$ and $\sigma^2_{\tilde{d}^{(R)}}$ are the variances of $d_n^{(R)}$ and $\tilde{d}_{n'}^{(R)}$, respectively. From (7), it can be concluded that, in the frequency-domain, the PA can be viewed as a linear system with a fixed gain $K_0^{(R)}$ and an additive uncorrelated noise of variance $\sigma^2_{d^{(R)}}$. For certain nonlinear functions $F(\cdot)$, there are analytical expressions for $K_0^{(R)}$ and $\sigma^2_{d^{(R)}}$ [12, 9].

$$\gamma_n^{(SRD)} = \frac{\left|h_n^{(RD)}h_n^{(SR)}\right|^2\left|K_0^{(R)}\right|^2 P_r P_s}{\sigma_v^2\left(\left|h_n^{(SR)}\right|^2 P_s + \left|h_n^{(RD)}\right|^2\left|K_0^{(R)}\right|^2 P_r + \sigma_v^2\right) + \left|h_n^{(RD)}\right|^2 \sigma_{d^{(R)}}^2\left(\left|h_n^{(SR)}\right|^2 P_s + \sigma_v^2\right)} \quad (16)$$

$$\gamma_n^{(SRD)} = \frac{\gamma_n^{(SR)}\gamma_n^{(RD)}\gamma^{(PA)}}{\gamma_n^{(SR)}\gamma_n^{(RD)} + \gamma_n^{(SR)}\gamma^{(PA)} + \gamma_n^{(RD)}\gamma^{(PA)} + \gamma_n^{(RD)} + \gamma^{(PA)}} \quad (17)$$

The signal received by the destination at the n^{th} subcarrier during the transmission stage 2 in the frequency-domain is given by:

$$x_n^{(RD)} = h_n^{(RD)}u_n^{(R)} + v_n^{(RD)} \quad (9)$$

where $h_n^{(RD)}$ is the CFR at the n^{th} subcarrier of the relay-destination channel and $v_n^{(RD)}$ is the corresponding noise component.

Substituting (2) and (7) into (9), we get:

$$\begin{aligned} x_n^{(RD)} &= h_n^{(RD)}K_0^{(R)}g_n h_n^{(SR)}\sqrt{P_s}\, s_n \\ &\quad + h_n^{(RD)}K_0^{(R)}g_n v_n^{(SR)} + h_n^{(RD)}d_n^{(R)} + v_n^{(RD)}, \end{aligned} \quad (10)$$

or, equivalently:

$$x_n^{(RD)} = h_n^{(SRD)}s_n + v_n^{(SRD)}, \quad (11)$$

where

$$h_n^{(SRD)} = h_n^{(RD)}K_0^{(R)}g_n h_n^{(SR)}\sqrt{P_s} \quad (12)$$

and

$$v_n^{(SRD)} = h_n^{(RD)}K_0^{(R)}g_n v_n^{(SR)} + h_n^{(RD)}d_n^{(R)} + v_n^{(RD)}. \quad (13)$$

It is worth noting that $\mathbb{E}[v_n^{(SRD)}v_n^{(SD)^*}] = 0$, $\mathbb{E}[d_n^{(R)}s_n^*] = 0$ and $\mathbb{E}[v_n^{(SRD)}s_n^*] = 0$.

From (2), (7) and (9), note that the source-relay-destination link can be viewed as a series-cascade of three linear channels. In other words, nonlinear PA acts as another linear relaying channel, placed between the relay and the destination.

3. OUTAGE ANALYSIS

In this section, an expression for the outage probability of the nonlinear cooperative AF OFDM system presented in Section 2 is derived assuming that the wireless channels have Rayleigh fading. Herein, we denote by $f_X(\cdot)$ and $F_X(\cdot)$ the probability density function (PDF) and cumulative distribution function (CDF) of the random variable X, respectively.

3.1. Instantaneous Signal-to-Noise Ratio

The instantaneous SNR after the NLD-aware MRC is given by $\gamma_n = \gamma_n^{(SD)} + \gamma_n^{(SRD)}$, with $\gamma_n^{(SD)} = |h_n^{(SD)}|^2 P_s/\sigma_v^2$ and $\gamma_n^{(SRD)} = |h_n^{(SRD)}|^2/\sigma_{v'_n}^2$, where $\sigma_{v'_n}^2$ is the variance of $v_n^{(SRD)}$ given by:

$$\sigma_{v'_n}^2 = \sigma_v^2\left(1 + |h_n^{(RD)}|^2|K_0^{(R)}|^2 g_n^2\right) + |h_n^{(RD)}|^2\sigma_{d^{(R)}}^2. \quad (14)$$

In [8], this MRC receiver is called NLD-aware MRC, as the receiver must be aware of the NLD to carry out the MRC.

From (12) and (14), it follows that:

$$\gamma_n^{(SRD)} = \frac{\left|h_n^{(RD)}h_n^{(SR)}\right|^2 g_n^2\left|K_0^{(R)}\right|^2 P_s}{\sigma_v^2\left(1 + |h_n^{(RD)}|^2\left|K_0^{(R)}\right|^2 g_n^2\right) + |h_n^{(RD)}|^2\sigma_{d^{(R)}}^2}. \quad (15)$$

Then, using (4), we obtain (16) at the top of the page.

By its turn, defining the following wireless links instantaneous SNRs: $\gamma_n^{(SR)} = |h_n^{(SR)}|^2 P_s/\sigma_v^2$ and $\gamma_n^{(RD)} = |h_n^{(RD)}|^2|K_0^{(R)}|^2 P_r/\sigma_v^2$, and the instantaneous SNR at the output of the PA as: $\gamma^{(PA)} = |K_0^{(R)}|^2 P_r/\sigma_{d^{(R)}}^2$, (16) can be rewritten as (17). Considering the PA as a linear channel, (17) corresponds to the instantaneous SNR of a three-hop cooperative AF system. Indeed, by defining:

$$\gamma_n^{(PA,RD)} = \frac{\gamma^{(PA)}\gamma_n^{(RD)}}{\gamma^{(PA)} + \gamma_n^{(RD)}}, \quad (18)$$

(17) can be expressed as:

$$\gamma_n^{(SRD)} = \frac{\gamma_n^{(SR)}\gamma_n^{(PA,RD)}}{\gamma_n^{(SR)} + \gamma_n^{(PA,RD)} + 1}. \quad (19)$$

Note that (19) corresponds to the instantaneous SNR of a two-hop cooperative AF system with individual link SNRs given by $\gamma^{(SR)}$ and $\gamma_n^{(PA,RD)}$, and a variable gain given by (4), while (18) corresponds to the instantaneous SNR of a two-hop system with individual link SNRs given by $\gamma^{(PA)}$ and $\gamma_n^{(RD)}$, and a relay gain given by $1/|K_0^{(R)}|$ [13].

The SNR γ_n can be approximated by the following upper bound:

$$\gamma_n = \gamma_n^{(SD)} + \min(\gamma_n^{(SR)}, \gamma^{(PA)}, \gamma_n^{(RD)}). \quad (20)$$

It is noteworthy that the approximation above is adopted in many previous works that investigate multihop relaying systems [14, 13]. As it will be shown in the simulation results section, such an approximation also applies to our work, being reasonably accurate for low and high SNRs.

3.2. Outage Probability

Defining the outage probability of the n^{th} subcarrier of the considered OFDM system as: $P_n^{(out)}(\gamma) = \text{Prob}\,(\gamma_n < \gamma)$, for $\gamma_n, \gamma \geq 0$, we can write:

$$\begin{aligned} P_n^{(out)}(\gamma) &= \text{Prob}\left(\gamma_n^{(SD)} + \gamma_n^{(min)} < \gamma\right) \\ &= \int_0^{+\infty} f_{\gamma_n^{(SD)}}(x)\left[\int_0^{\gamma - x} f_{\gamma_n^{(min)}}(y)\,dy\right]dx \\ &= \int_0^{+\infty} f_{\gamma_n^{(SD)}}(x)F_{\gamma_n^{(min)}}(\gamma - x)\,dx, \end{aligned} \quad (21)$$

for $\gamma \geq x$, where $\gamma_n^{(min)} = \min(\gamma_n^{(SR)}, \gamma^{(PA)}, \gamma_n^{(RD)})$ and:

$$F_{\gamma_n^{(min)}}(\gamma) = 1 - \left[1 - F_{\gamma_n^{(SR)}}(\gamma)\right]\left[1 - F_{\gamma^{(PA)}}(\gamma)\right]\left[1 - F_{\gamma_n^{(RD)}}(\gamma)\right]. \quad (22)$$

Let us assume that the instantaneous SNRs of the wireless links are exponentially distributed (Rayleigh fading), that is: $F_{\gamma_n^{(SR)}}(\gamma) = [1-\exp(-\gamma/\bar{\gamma}^{(SR)})]u(\gamma)$ and $F_{\gamma_n^{(RD)}}(\gamma) = [1-\exp(-\gamma/\bar{\gamma}^{(RD)})]$ $u(\gamma)$, where $\bar{\gamma}^{(SR)}$ and $\bar{\gamma}^{(RD)}$ denotes the standard deviation of $\gamma_n^{(SR)}$ and $\gamma_n^{(RD)}$, respectively, and $u(\cdot)$ is the unit step function. Moreover, let us consider that the PA SNR is fixed, that is, $F_{\gamma^{(PA)}}(\gamma) = u(\gamma - \bar{\gamma}^{(PA)})$, where $\bar{\gamma}^{(PA)} = |K_0^{(R)}|^2 P_r/\sigma^2_{d(R)}$. Thus, (22) can be re-expressed as:

$$\begin{aligned} F_{\gamma_n^{(min)}}(\gamma) &= 1 - \left[u(-\gamma) + e^{\frac{-\gamma}{\bar{\gamma}^{(SR)}}} u(\gamma)\right] \left[u(-\gamma) + e^{\frac{-\gamma}{\bar{\gamma}^{(RD)}}} u(\gamma)\right] u\left(\bar{\gamma}^{(PA)} - \gamma\right) \\ &= u(\gamma) - e^{-\left(\frac{\gamma}{\bar{\gamma}^{(SR)}} + \frac{\gamma}{\bar{\gamma}^{(RD)}}\right)} u(\gamma) u\left(\bar{\gamma}^{(PA)} - \gamma\right). \end{aligned} \quad (23)$$

Substituting (23) into (25), we get:

$$P_n^{(out)}(\gamma) = \int_0^{+\infty} f_{\gamma_n^{(SD)}}(x) \left[u(\gamma - x) - e^{-\left(\frac{\gamma - x}{\bar{\gamma}^{(SR)}} + \frac{\gamma - x}{\bar{\gamma}^{(RD)}}\right)} u(\gamma - x) u\left(\bar{\gamma}^{(PA)} - \gamma + x\right)\right] dx. \quad (24)$$

Assuming that $\gamma_n^{(SD)}$ is exponentially distributed with standard deviation $\bar{\gamma}^{(SD)}$, it follows:

$$P_n^{(out)}(\gamma) = F_{\gamma_n^{(SD)}}(\gamma) - \frac{1}{\bar{\gamma}^{(SD)}} e^{-\left(\frac{\gamma}{\bar{\gamma}^{(SR)}} + \frac{\gamma}{\bar{\gamma}^{(RD)}}\right)} \int_{\max(\gamma - \bar{\gamma}^{(PA)}, 0)}^{\gamma} e^{x/\bar{\gamma}_M} dx, \quad (25)$$

where $\bar{\gamma}_M = \left(-\frac{1}{\bar{\gamma}^{(SD)}} + \frac{1}{\bar{\gamma}^{(SR)}} + \frac{1}{\bar{\gamma}^{(RD)}}\right)^{-1}$.

Eq. (25) can be rewritten as:

$$\begin{aligned} P_n^{(out)}(\gamma) &= F_{\gamma_n^{(SD)}}(\gamma) - \frac{\bar{\gamma}_M}{\bar{\gamma}^{(SD)}} e^{-\left(\frac{\gamma}{\bar{\gamma}^{(SR)}} + \frac{\gamma}{\bar{\gamma}^{(RD)}}\right)} \left[e^{\frac{\gamma}{\bar{\gamma}_M}} - e^{\frac{\max(\gamma - \bar{\gamma}^{(PA)}, 0)}{\bar{\gamma}_M}}\right] \\ &= 1 - e^{-\frac{\gamma}{\bar{\gamma}^{(SD)}}} - \frac{\bar{\gamma}_M}{\bar{\gamma}^{(SD)}} e^{-\left(\frac{\gamma}{\bar{\gamma}^{(SR)}} + \frac{\gamma}{\bar{\gamma}^{(RD)}}\right)} \left[1 - e^{-\frac{\min\left(\gamma, \bar{\gamma}^{(PA)}\right)}{\bar{\gamma}_M}}\right] e^{\frac{\gamma}{\bar{\gamma}_M}}, \end{aligned} \quad (26)$$

which leads to:

$$P^{(out)}(\gamma) = 1 - \left[1 + \frac{\bar{\gamma}_M}{\bar{\gamma}^{(SD)}} \left(1 - e^{-\frac{\min\left(\gamma, \bar{\gamma}^{(PA)}\right)}{\bar{\gamma}_M}}\right)\right] e^{-\frac{\gamma}{\bar{\gamma}^{(SD)}}}. \quad (27)$$

As the outage probability in (27) does depend on the subcarrier, the index n was omitted from $P_n^{(out)}(\gamma)$.

3.3. Discussion

The first interesting remark about (27) is that the outage probability depends on the nonlinear PA parameters only for $\gamma > \bar{\gamma}^{(PA)}$. To have a better understanding of the impact of the PA nonlinearity, let us consider that $\bar{\gamma}^{(SD)} = \bar{\gamma}^{(SR)} = \bar{\gamma}^{(RD)} = \bar{\gamma}$. In this case, (27) becomes:

$$P^{(out)}(\gamma) = 1 - 2e^{-\frac{\gamma}{\bar{\gamma}}} + e^{-\frac{\min\left(\gamma, \bar{\gamma}^{(PA)}\right) + \gamma}{\bar{\gamma}}}. \quad (28)$$

For a linear PA, we have:

$$P^{(out)}(\gamma) = 1 - 2e^{-\frac{\gamma}{\bar{\gamma}}} + e^{-2\frac{\gamma}{\bar{\gamma}}}. \quad (29)$$

Note that when $\gamma \leq \bar{\gamma}^{(PA)}$, (28) and (29) become equivalent, and when $\gamma > \bar{\gamma}^{(PA)}$, we have:

$$P^{(out)}(\gamma) = 1 - 2e^{-\frac{\gamma}{\bar{\gamma}}} + e^{-\frac{\bar{\gamma}^{(PA)} + \gamma}{\bar{\gamma}}}. \quad (30)$$

By comparing (30) and (29), one can see that for $\gamma > \bar{\gamma}^{(PA)}$, $P^{(out)}(\gamma)$ in (29) is smaller than in (30). This means that the PA nonlinearity increases the outage probability for high SNR thresholds γ, but it does not change the outage probability for small γ.

Moreover, when there is no direct path between source and destination, the outage probability is as follows: $P^{(out)}(\gamma) = F_{\gamma_n^{(min)}}(\gamma)$, given in (23).

4. SIMULATION RESULTS

In this section, the validity of the outage probability expression developed in Section 3 is evaluated by means of simulations. An AF cooperative OFDM system with frequency-selective Rayleigh fading channels has been considered for the simulations, with the relay PA being modeled by a soft clipper (soft limiter). Closed-form expressions for $K_0^{(R)}$ and $\sigma^2_{d(R)}$ are given in the Appendix. The channel impulse responses have 4 independent taps and the length of the cyclic prefixes is equal to 4 sampling periods. The results were obtained with $N = 256$ subcarriers and 16-QAM transmitted signals, via Monte Carlo simulations. It is assumed that the channels change every 10 symbol periods and that the source-destination channel has a mean power 10dB lower than the source-relay and relay-destination channels. Moreover, it is assumed that the destination knows perfectly all the channels and that the relay knows perfectly the source-relay channel, allowing the use of the variable gain given in (4). An input back-off (IBO) of 5dB is used at the relay, which leads to $\bar{\gamma}^{(PA)} = 23.5dB$. Besides, we consider that $P_s + P_r = 1$

Fig. 2 shows the outage probability versus the mean SNR at the source-destination link, the outage probability being obtained by means of simulations and using (27), for various values of γ. It can be noted that the theoretical and simulated curves are very close, specially for low and high values of γ. Moreover, by the slopes of the curves, it can be concluded that the system diversity is higher when $\gamma < \bar{\gamma}^{(PA)} = 23.5dB$. This is due to the fact the NLD is more significant for high channel SNRs than for lower SNRs.

Fig. 3 shows $P^{(out)}(\gamma)$ versus the mean SNR at the source-destination link, $P^{(out)}(\gamma)$ being obtained by means of simulations and using (27), considering linear and nonlinear PAs, for various values of γ. It can be viewed that for $\gamma > \bar{\gamma}^{(PA)}$, the nonlinear PA provides outage probabilities significantly higher than the linear PA. However, when the value of γ decreases, the difference between the simulated outage probability with the linear and the nonlinear PA

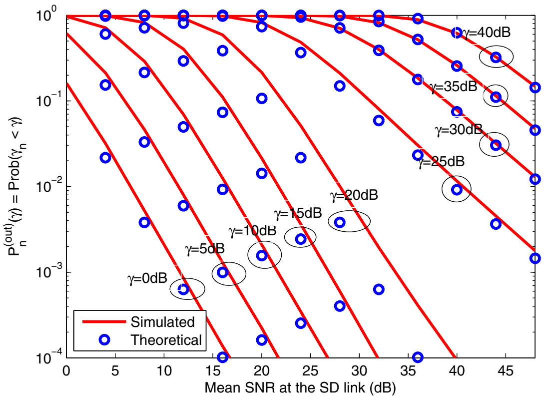

Fig. 2. $P^{(out)}(\gamma)$ versus the mean SNR for various values of γ.

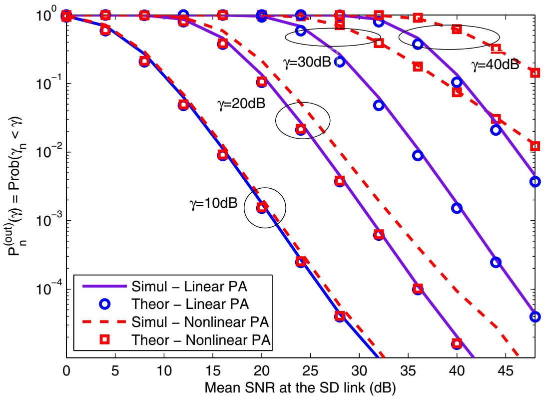

Fig. 3. $P^{(out)}(\gamma)$ versus the mean SNR for linear and nonlinear PAs.

becomes smaller. Moreover, for $\gamma < \bar{\gamma}^{(PA)}$, the theoretical outage probabilities obtained with the linear and the nonlinear PA are equal. That allows to concluded that the PA nonlinearity affects the system outage probability only for high values of γ.

5. CONCLUSION

A theoretical outage analysis of an AF cooperative OFDM systems with nonlinear PAs has been developed in this paper. An outage probability expression has been derived by approximating the instantaneous SNR of the MRC output by an upper bound. Simulation results have shown that the developed outage probability expression has good accuracy and that the nonlinear PA decreases the system diversity only for high SNR thresholds. In future works, the analysis of this paper should be extended to more general channel fading models and to multiple relays.

A. APPENDIX - SOFT CLIPPING MODEL

The soft clipper (or soft limiter) model for a PA is defined as [12, 9]:

$$\tilde{u}_{n'}^{(R)} = \begin{cases} \tilde{q}_{n'}^{(R)}, & \text{if } |\tilde{q}_{n'}^{(R)}| \leq A_{sat}, \\ e^{\jmath \angle \tilde{q}_{n'}^{(R)}}, & \text{if } |\tilde{q}_{n'}^{(R)}| > A_{sat}, \end{cases} \tag{31}$$

A_{sat} being the saturation or clipping level (maximum output amplitude). In this case, $K_0^{(R)}$ and $\sigma^2_{d(R)}$ are respectively given by:

$$K_0^{(R)} = 1 - e^{\left(-A_{sat}^2/P_s\right)} + \frac{\sqrt{\pi}\, A_{sat}}{2\sqrt{P_s}} \operatorname{erfc}\left(\frac{A_{sat}}{\sqrt{P_s}}\right) \tag{32}$$

and

$$\sigma^2_{d(R)} = P_s \left(1 - e^{\left(-A_{sat}^2/P_s\right)} - |K_0^{(R)}|^2\right). \tag{33}$$

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