Blind MIMO channel identification using cumulant tensor decomposition

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Abstract In this paper, we exploit the symmetry properties of fourth-order cumulants to develop a new blind identification algorithm for multiple-input multiple-output (MIMO) instantaneous channels. The proposed algorithm utilizes the Parallel Factor (Parafac) decomposition of the 4th-order cumulant tensor by solving a single-step (SS) least squares (LS) problem. This approach is shown to hold for channels with more sources than sensors. A simplified approach using a reduced-order tensor is also discussed. Computer simulations are provided to illustrate the performance of the proposed identification algorithms.

I. The blind identification problem

Cumulants of order higher than two can be viewed as tensors with a highly symmetrical structure. For about two decades, exploiting the cumulant symmetries with a tensor formalism has been an important research topic. The Parallel Factor (Parafac) decomposition of a Pth-order tensor with rank Q consists in decomposing it into a sum of Q rank-one tensors [1]. The key-point in the use of the Parafac decomposition is its uniqueness property, which can be assured under simple conditions, stated by the Kruskal Theorem [2]. Parafac does not induce neither rotational ambiguities nor orthogonality constraints, as it is the case with matrix singular value decomposition. For that reason, the use of cumulant tensor factorization allows for avoiding the *pre-whitening* step, a time-consuming operation responsible for increased estimation errors [3].

The alternating least squares (ALS) algorithm consists in fitting a Pth-order Parafac model by iteratively minimizing, in an alternate way, P least squares (LS) cost functions. Our focus in this paper is to exploit the redundancies of the parallel factors of the 4th-order cumulant tensor in the minimization problem in order to develop a new blind channel identification (BCI) algorithm. We consider the problem of blind multiple-input multiple-output (MIMO) channel (mixture) identification in the context of a multiuser system characterized by instantaneous complex-valued channels. We introduce new algorithms based on the Parafac decomposition of cumulant tensors, as an extension of the algorithms proposed in [4] for the case of single-input single-output FIR channels. Our main contribution consists in exploiting the redundancies in the Parafac components to estimate the channel matrix by solving a single LS minimization problem. This approach greatly simplifies the estimation problem and allows us to introduce a new blind MIMO channel estimation algorithm based on a single-step (SS) LS optimization procedure.

During the two last decades, several BCI methods making use of the redundancies in the 4th-order cumulants have been proposed [5]. Such approaches include, for instance, the popular joint approximate diagonalization of eigenmatrices (JADE) algorithm [6], which is based on second and fourth-order statistics. It is now well-known that in the case of linear mixtures the BCI problem is closely related to the (simultaneous) diagonalization of symmetric cumulant tensors [7]. Important connections have been recently established between different joint diagonalization criteria and the canonical tensor decomposition [8], [9]. The ICAR algorithm proposed in [10] applies independent component analysis (ICA) using the redundancies in the quadricovariance in order to estimate overdetermined mixtures (more sensors than sources). More recently, [11] proposed the Fourth-order-only blind identification (FOOBI) algorithm, in which the canonical components of the 4th-order cumulant tensor decomposition are obtained from a simultaneous matrix diagonalization by congruence transformation. In both cases, breaking the problem into two optimization procedures remain necessary to reach a final solution. Our contribution, on the other hand, is based on the solution of a single minimization problem. Computer simulations illustrate the performance gains that our method provides with respect to other existing solutions.

Making use of some tensor properties, we also show that under certain conditions our algorithm is able to identify channels with more sources than sensors. Other approaches for BCI in the case of under-determined mixtures can be found in the literature, including the use of the virtual array concept [12] and a Parafac-based frequency-domain framework for MIMO BCI [3]. The FOBIUM method proposed in [13] can be viewed as an extension of the well-known SOBI algorithm [14] to the 4th-order allowing for the estimation of underdetermined mixtures, without SOS-based pre-whitening. The FOOBI algorithm [11] also treats the underdetermined case but with weaker uniqueness conditions and hence, theoretically better identifiability properties.

The rest of this paper is organized as follows: in section II, we introduce the signal model and express the tensor of output cumulants as a Parafac model; the uniqueness and identifiability issues are also discussed; in section III, we describe the single-

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step LS approach giving rise to our Parafac-based blind MIMO channel identification (PBMCI) algorithms; section IV presents some simulation results and, in section V, we draw some conclusions and discuss perspectives.

II. MIMO CHANNEL AND CUMULANT TENSORS

Let us consider an instantaneous MIMO channel in which the output signal vector $\mathbf{y}(n) \in \mathbb{C}^M$ is given by:

$$\mathbf{y}(n) = \mathbf{H}\mathbf{s}(n) + \boldsymbol{v}(n), \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{M \times Q}$ is the MIMO channel matrix with elements $h_{m,q}$, representing a Rayleigh flat fading propagation environment, i.e. the channel coefficients are complex constants with real and imaginary parts driven from a continuous Gaussian distribution. This assumption allows us to say that $\mathbf{H}$ is full k-rank with probability one, which means that $k_{\mathbf{H}} = r_{\mathbf{H}} = \min(M, Q)$. The source signals $s_q(n)$ are assumed to be stationary, ergodic and mutually independent with symmetric distribution and known non-zero kurtosis γ_{4,s_q}. The additive noise $\boldsymbol{v}(n) \in \mathbb{C}^{M \times 1}$ is driven from a Gaussian distribution with zero-mean, independent from the input signals. Assuming that γ_{4,s_q} is known is usual in the context of telecommunication systems, where the source modulation schemes are generally known by the receiver. However this assumption is not necessary and could be relaxed. Note that we do not impose any constraints to the sign of the source kurtoses.

Under the above assumptions, we address the problem of blindly estimating the channel matrix $\mathbf{H}$, up to column scaling and permutation, using 4th-order output cumulants only. Defining the 4th-order spatial cumulants as $C_{4,y}(i,j,k,l) \triangleq \textit{cum}\,[y_i^*(n), y_j(n), y_k^*(n), y_l(n)]$, we get:

$$C_{4,y}(i,j,k,l) = \sum_{q=1}^{Q} \gamma_{4,s_q} h_{i,q}^* h_{j,q} h_{k,q}^* h_{l,q}. \tag{2}$$

Equation (2) corresponds to the scalar representation of the 4th-order tensor $\mathcal{C}^{(4,y)} \in \mathbb{C}^{M \times M \times M \times M}$, so that $\mathcal{C}^{(4,y)}$ can be viewed as a sum of Q rank-1 4th-order tensors, as follows:

$$\mathcal{C}^{(4,y)} = \sum_{q=1}^{Q} \mathbf{H}_{\cdot q}^* \circ \mathbf{H}_{\cdot q} \circ \mathbf{H}_{\cdot q}^* \circ (\gamma_{4,s_q} \mathbf{H}_{\cdot q}) \tag{3}$$

where $\circ$ denotes the outer product. Equivalently, we can unfold $\mathcal{C}^{(4,y)}$ into a 2D representation $\mathbf{T} \in \mathbb{C}^{M^3 \times M}$, defined as the matrix in which the element $C_{4,y}(i,j,k,l)$ is placed at column l and row $(i-1)M^2 + (j-1)M + k$. Equation (3) is the 4th-order Parafac representation of the tensor $\mathcal{C}^{(4,y)}$. Denoting the mth canonical basis vector of $\mathbb{R}^M$ by $\mathbf{e}_m^{(M)}$, we have[1]:

$$\mathbf{T} = \sum_{i,j,k=1}^{M} C_{4,y}(i,j,k,l) \left(\mathbf{e}_i^{(M)} \diamond \mathbf{e}_j^{(M)} \diamond \mathbf{e}_k^{(M)} \right) \mathbf{e}_l^{(M)\mathsf{T}} \tag{4}$$

[1] In practice, we form the column $\mathbf{T}_{\cdot l}$, $l \in [1, M]$, with the elements $C_{4,y}(i,j,k,l)$ by varying the indices $i,j,k \in [1, M]$ in nested loops with k corresponding to the innermost (fastest) lop and i to the outermost (slowest) one.

where $\diamond$ denotes the Khatri-Rao product (column-wise Kronecker product) and we use the fact that $\mathbf{e}_i^{(I)} \diamond \mathbf{e}_j^{(J)} = \mathbf{e}_{(i-1)J+j}^{(IJ)}$. Replacing (2) into equation (4), we get:

$$\begin{aligned} \mathbf{T} &= \sum_{q=1}^{Q} (\mathbf{H}_{\cdot q}^* \diamond \mathbf{H}_{\cdot q} \diamond \mathbf{H}_{\cdot q}^*)(\gamma_{4,s_q} \mathbf{H}_{\cdot q})^{\mathsf{T}} \\ &= (\mathbf{H}^* \diamond \mathbf{H} \diamond \mathbf{H}^*)(\mathbf{H}\mathbf{\Gamma})^{\mathsf{T}} \quad \in \mathbb{C}^{M^3 \times M}, \end{aligned} \tag{5}$$

where $\mathbf{\Gamma} = \textit{Diag}\left(\gamma_{4,s_1}, \ldots, \gamma_{4,s_Q}\right)$. Rewriting (5) as $\mathbf{T} = (\bar{\mathbf{A}} \diamond \bar{\mathbf{B}} \diamond \bar{\mathbf{C}})\bar{\mathbf{D}}^{\mathsf{T}}$, the Parafac decomposition given in (3) is said to be essentially unique if the components $\{\bar{\mathbf{A}}, \bar{\mathbf{B}}, \bar{\mathbf{C}}, \bar{\mathbf{D}}\}$ are such that:

$$\begin{aligned} \bar{\mathbf{A}} &= \mathbf{H}^* \mathbf{\Lambda}_1 \mathbf{\Pi}, & \bar{\mathbf{B}} &= \mathbf{H} \mathbf{\Lambda}_2 \mathbf{\Pi}, \\ \bar{\mathbf{C}} &= \mathbf{H}^* \mathbf{\Lambda}_3 \mathbf{\Pi}, & \bar{\mathbf{D}} &= \mathbf{H} \mathbf{\Gamma} \mathbf{\Lambda}_4 \mathbf{\Pi}, \end{aligned} \tag{6}$$

with $\mathbf{\Pi}$ a permutation matrix and $\mathbf{\Lambda}_p$, $p \in \{1,2,3,4\}$, diagonal scaling matrices satisfying $\prod_{p=1}^{4} \mathbf{\Lambda}_p = \mathbf{I}_Q$ [15].

A. Uniqueness and identifiability

A sufficient uniqueness condition for the Parafac decomposition of a 3rd-order tensor was established by Kruskal in [16] and extended to the case of a Pth-order tensor in [2], as follows: $\sum_{p=1}^{P} k_{\mathbf{A}^{(p)}} \geq 2Q + (P-1)$, where Q is the tensor rank and $k_{\mathbf{A}^{(p)}}$ is the k-rank of the Parafac component $\mathbf{A}^{(p)}$. For the 4th-order tensor (3), since $k_{\mathbf{H}} = r_{\mathbf{H}} = \min(M, Q)$, the Kruskal uniqueness condition reduces to:

$$4k_{\mathbf{H}} \geq 2Q + 3. \tag{7}$$

The two following cases can be considered:

- $M \geq Q$ (over-determined channel): we get $r_{\mathbf{H}} = Q$ and (7) yields $Q \geq 3/2$, i.e. $Q > 1$. There is no further constraint on the number of sensors.
- $M < Q$ (under-determined channel): in this case $r_{\mathbf{H}} = M$ and hence equation (7) becomes:

$$Q \leq \frac{4M - 3}{2}.$$

Table I shows the maximum number of sources that can be theoretically identified for a given number of receive antennas (varying from $M = 2$ to $M = 7$). When this condition is satisfied, the channel matrix $\mathbf{H}$ can be determined, up to column permutation and scaling ambiguities.

B. Reduced-order tensor

By combining the 3D slices associated with one of the four-dimensions of tensor $\mathcal{C}^{(4,y)}$, we can build a reduced-order tensor of 4th-order cumulants. Without loss of generality, we eliminate the index k in (2) to get a 3rd-order tensor $\mathcal{C}^{(3,y)}$ with scalar representation given by $c_{ijl}^{(3)} = \sum_k C_{4,y}(i,j,k,l)$. This yields:

$$\mathcal{C}^{(3,y)} = \sum_{q=1}^{Q} \mathbf{H}_{\cdot q}^* \circ \mathbf{H}_{\cdot q} \circ \left(\gamma_{4,s_q} \mathbf{H}_{\cdot q} \sum_{k=1}^{M} h_{kq}^* \right), \tag{8}$$

TABLE I
MAXIMUM NUMBER OF IDENTIFIABLE SOURCES

	$M =$	2	3	4	5	6	7
Using $\mathcal{C}^{(4,y)}$	$Q \leq$	2	4	6	8	10	12
Using $\mathcal{C}^{(3,y)}$	$Q \leq$	2	3	5	6	8	9

which allows for a straightforward unfolded representation $\mathbf{T} \in \mathbb{C}^{M^2 \times M}$, defined as the matrix in which the element $c^{(3)}_{ijl}$ is placed at column l and row $(i-1)M + j$, so that:

$$\mathbf{T} = \sum_{i,j=1}^{M} c^{(3)}_{i,j,l} \left(\mathbf{e}_i^{(M)} \diamond \mathbf{e}_j^{(M)}\right) \mathbf{e}_l^{(M)\mathsf{T}} \tag{9}$$

and hence

$$\begin{aligned} \mathbf{T} &= \sum_{q=1}^{Q} (\mathbf{H}^*_{\cdot q} \diamond \mathbf{H}_{\cdot q})(\gamma_{4,s_q} \mathbf{H}_{\cdot q} \sum_{k=1}^{M} h^*_{kq})^{\mathsf{T}} \\ &= (\mathbf{H}^* \diamond \mathbf{H})(\mathbf{H}\mathbf{\Gamma}\mathbf{\Delta})^{\mathsf{T}} \quad \in \mathbb{C}^{M^2 \times M}, \end{aligned} \tag{10}$$

where $\mathbf{\Delta}$ is a diagonal matrix given by:

$$\mathbf{\Delta} = \sum_{k=1}^{M} D_k\left(\mathbf{H}^*\right). \tag{11}$$

Rewriting the above equation as $\mathbf{T} = (\bar{\mathbf{A}} \diamond \bar{\mathbf{B}})\bar{\mathbf{C}}^{\mathsf{T}}$, the Parafac decomposition given in (8) is said to be essentially unique if the matrices $\{\bar{\mathbf{A}}, \bar{\mathbf{B}}, \bar{\mathbf{C}}\}$ are such that:

$$\bar{\mathbf{A}} = \mathbf{H}^* \mathbf{\Lambda}_1 \mathbf{\Pi}, \quad \bar{\mathbf{B}} = \mathbf{H}\mathbf{\Lambda}_2\mathbf{\Pi} \quad \text{and} \quad \bar{\mathbf{C}} = \gamma_{4,s}\mathbf{H}\mathbf{\Gamma}\mathbf{\Delta}\mathbf{\Lambda}_3\mathbf{\Pi}, \tag{12}$$

where $\mathbf{\Pi}$ and $\mathbf{\Lambda}_p$, $p \in \{1,2,3\}$, are defined as in (6) with $\mathbf{\Lambda}_1\mathbf{\Lambda}_2\mathbf{\Lambda}_3 = \mathbf{I}_Q$.

As $\mathbf{H}$ is assumed to be a constant matrix with elements driven from a continuous Gaussian distribution, we can conclude that all the diagonal entries of $\mathbf{\Delta}$ are nonzero with probability one. In this case, the Kruskal uniqueness condition becomes $3r_{\mathbf{H}} \geq 2Q + 2$, which yields $Q \geq 2$ for $M \geq Q$ and $Q \leq (3M-2)/2$ when $M < Q$. Table I shows the maximum number of sources that can be separated using tensor $\mathcal{C}^{(3,y)}$ for a given number of receive antennas ($M \in [2,7]$). Under this condition, the Parafac components of $\mathcal{C}^{(3,y)}$ are given by (12). Note that tensor $\mathcal{C}^{(3,y)}$ is composed of 4th-order statistics only.

III. SINGLE-STEP LS ALGORITHMS

We propose to estimate the MIMO channel matrix $\mathbf{H}$ by exploiting the symmetry properties of the 4th-order cumulant tensors $\mathcal{C}^{(4,y)}$ and $\mathcal{C}^{(3,y)}$. Equations (5) and (10) yield unfolded representations of these tensors, clearly showing the dependence between their Parafac components. The channel parameters can then be estimated using the fact that all the Parafac components depend on the channel matrix.

A. Single-step LS channel estimation using $\mathcal{C}^{(4,y)}$

Equation (5) enables us to estimate the MIMO channel matrix by minimizing a single LS cost function that is written as follows:

$$\psi(\hat{\mathbf{H}}_{r-1}, \mathbf{H}) \triangleq \left\|\mathbf{T} - (\hat{\mathbf{H}}^*_{r-1} \diamond \hat{\mathbf{H}}_{r-1} \diamond \hat{\mathbf{H}}^*_{r-1})(\mathbf{H}\mathbf{\Gamma})^{\mathsf{T}}\right\|_F^2 \tag{13}$$

where $\|\cdot\|_F$ denotes the Frobenius norm and r is the iteration number. Iteratively minimizing $\psi(\hat{\mathbf{H}}_{r-1}, \mathbf{H})$ yields the following LS solution:

$$\begin{aligned} \hat{\mathbf{H}}_r^{\mathsf{T}} &\triangleq \arg\min_{\mathbf{H}} \psi(\hat{\mathbf{H}}_{r-1}, \mathbf{H}) \\ &= \mathbf{\Gamma}^{-1}(\hat{\mathbf{H}}^*_{r-1} \diamond \hat{\mathbf{H}}_{r-1} \diamond \hat{\mathbf{H}}^*_{r-1})^{\#}\mathbf{T} \end{aligned} \tag{14}$$

where $^{\#}$ denotes the matrix pseudoinverse and $\hat{\mathbf{H}}_0$ is initialized as a complex (column-normalized) $M \times Q$ Gaussian random matrix. At each iteration $r \geq 1$, we divide each column of $\hat{\mathbf{H}}_{r-1}$ by its respective norm before using it to compute the next estimate $\hat{\mathbf{H}}_r$. The algorithm is stopped when $\|\hat{\mathbf{H}}_r - \hat{\mathbf{H}}_{r-1}\|_F / \|\hat{\mathbf{H}}_r\|_F \leq \varepsilon$, where ε is an arbitrary small positive constant. The above described method will be referred to as the 4D Single-Step LS Parafac-based Blind MIMO Channel Identification (4D SS-LS PBMCI) algorithm.

B. Single-step LS channel estimation using $\mathcal{C}^{(3,y)}$

The SS approach can also be applied to the tensor $\mathcal{C}^{(3,y)}$ defined in (8). Analogously to the previous case, (10) yields a LS cost function that can be iteratively minimized, thus leading to:

$$\hat{\mathbf{H}}_r^{\mathsf{T}} = \mathbf{\Gamma}^{-1}\hat{\mathbf{\Delta}}^{-1}_{r-1}(\hat{\mathbf{H}}^*_{r-1} \diamond \hat{\mathbf{H}}_{r-1})^{\#}\mathbf{T}, \tag{15}$$

where $\hat{\mathbf{\Delta}}_{r-1}$ is given by (11) with $\hat{\mathbf{H}}_{r-1}$ replacing $\mathbf{H}$. Here again, $\hat{\mathbf{H}}_0$ is initialized as a complex column-normalized $M \times Q$ Gaussian random matrix. We will refer to this method as the 3D SS-LS PBMCI algorithm.

Under the conditions of Table I, irrespective of the tensor order ($\mathcal{C}^{(4,y)}$ or $\mathcal{C}^{(3,y)}$), any $\bar{\mathbf{H}}$ satisfying (5) or (10) is such that $\bar{\mathbf{H}} = \mathbf{H}\mathbf{\Lambda}\mathbf{\Pi}$, where $\mathbf{\Pi}$ is a permutation matrix and $\mathbf{\Lambda}$ is a diagonal matrix with unit-modulus diagonal elements.

IV. SIMULATION RESULTS

We simulated a *quasi-static* MIMO radio channel with coefficients driven from a Rayleigh distribution and assumed time-invariant within the duration of a time-slot ($N = 1000$ symbol periods). The channel coefficients vary independently between two successive time-slots. The results were averaged over $P = 300$ time-slots. In order to assess the performance of the proposed identification methods with QPSK modulated sources, we use the performance index defined in [17] as:

$$\begin{aligned} \xi(\mathbf{\Phi}^{\langle p \rangle}) \triangleq \frac{1}{2}\Bigg[&\left(\sum_i \left(\sum_j \frac{|\phi^{\langle p \rangle}_{i,j}|^2}{\max_\ell |\phi^{\langle p \rangle}_{i,\ell}|^2}\right) - 1\right) \\ &+ \left(\sum_j \left(\sum_i \frac{|\phi^{\langle p \rangle}_{i,j}|^2}{\max_\ell |\phi^{\langle p \rangle}_{\ell,j}|^2}\right) - 1\right)\Bigg], \end{aligned} \tag{16}$$

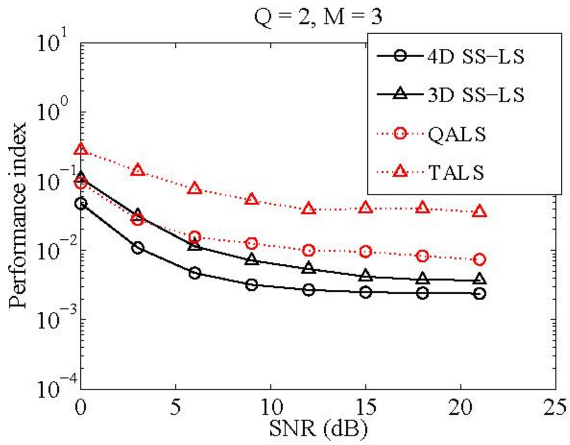

Fig. 1. Mean performance index vs. SNR: comparison between PBMCI algorithms.

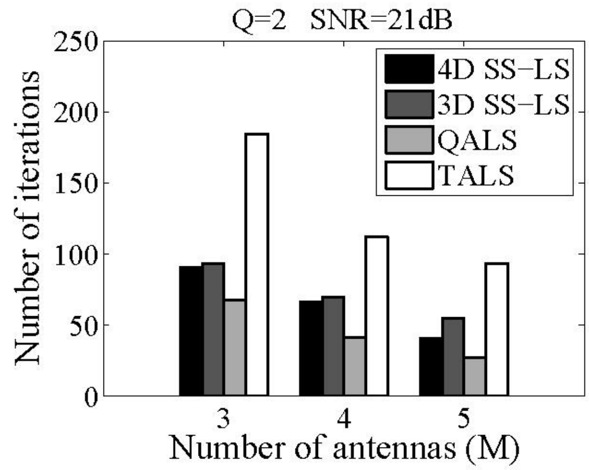

Fig. 2. Mean number of iterations for convergence: PBMCI algorithms.

where $\phi_{i,j}^{\langle p\rangle}$ are the entries of the matrix $\mathbf{\Phi}^{\langle p\rangle}$, defined as $\mathbf{\Phi}^{\langle p\rangle} = \mathbf{H}^{\#}\hat{\mathbf{H}}_{\langle p\rangle}$, where $\hat{\mathbf{H}}_{\langle p\rangle}$ is the channel estimate after convergence of the experiment $p \in [1, P]$. The index $\xi(\mathbf{\Phi}^{\langle p\rangle})$ approaches zero when the channel estimate approaches the desired solution, up to column scaling and permutation, indicating how close $\mathbf{\Phi}^{\langle p\rangle}$ is from a scaled permutation matrix. Equation (16) provides, therefore, a measure of the global level of interference rejection at the separator output, irrespective of permutation and scaling ambiguities.

We first evaluate the algorithms using the LS PBMCI approach. In fig. 1, we compare the proposed algorithms 4D SS-LS and 3D SS-LS with their ALS-based counterparts (QALS and TALS respectively). These curves show the mean performance index, obtained from (16), for $Q = 2$ users, with an array of $M = 3$ receive antennas, for a SNR range varying from 0 to 21 dB. Note that 4D SS-LS performs better than the other algorithms. Similar results have been obtained for other scenarii but are omitted here due to a lack of space. In fig. 2, we show the mean number of iterations needed for convergence of the four algorithms. As expected, the methods based on the 4th-order tensor converged faster than those based on the 3rd-order one. Note that, although 4D SS-LS takes more iterations to converge than QALS, its global computational complexity is much smaller than that of QALS, since it involves only one LS minimization per iteration instead of four.

In the sequel, we show some simulation results comparing the 4D SS-LS PBMCI algorithm with some of the most performing algorithms reported in the literature, including the classical JADE [6] algorithm, the FOOBI [11] and the ICAR [10] methods. Analogously to JADE, FOOBI and ICAR are also based on joint matrix diagonalization. FOOBI exploits the column-wise Kronecker structure of the quadricovariance matrix and induces uniqueness conditions weaker than those presented in this paper. Dealing with the same Hermitian cumulant matrix used by FOOBI, the ICAR method takes advantage of the redundancies in the 4th-order cumulant, but only works for over-determined mixtures. The extension of ICAR to higher orders (BIOME [12]) is able to identify under-determined mixtures, but it has not been included here as long as we limit our study to 4th-order cumulants. Also, SOBI [14] and its counterpart to the case of under-determined mixtures (FOBIUM [13]) have not been used because they are theoretically unable to deal with sources that have similar spectral contents.

From fig. 3, we conclude that the 4D SS-LS method performs quite well in terms of the performance index (16), for $Q = 2$ sources and $M = 3$ antennas. Note that the JADE algorithm gives the best results for high values of the SNR, but it is less performing than the other methods for an SNR lower than 12dB. On the other hand, ICAR and FOOBI perform nearly as well as 4D-SS, but they are also affected by the additive Gaussian noise for low SNR values. The results in fig. 4 were obtained with $Q = 3$ sources, indicating that our approach performs better than the other tested algorithms when increasing the number of sources for a fixed number of antennas. In conclusion, the proposed 4D SS-LS method seems to be a very interesting solution due to its simplicity and its robustness with respect to additive noise.

V. Conclusion

In this paper, the blind MIMO channel identification problem has been addressed in the context of a multiuser system characterized by instantaneous complex-valued channels. Parafac-based blind MIMO channel identification (PBMCI) algorithms have been proposed using the decomposition of tensors formed of 4th-order spatial cumulants by means of a single-step least squares (SS-LS) approach. This approach fully exploits the multidimensional nature of the cumulant tensor and has the advantage of avoiding any kind of pre-processing. Under certain conditions our algorithm is able to identify underdetermined

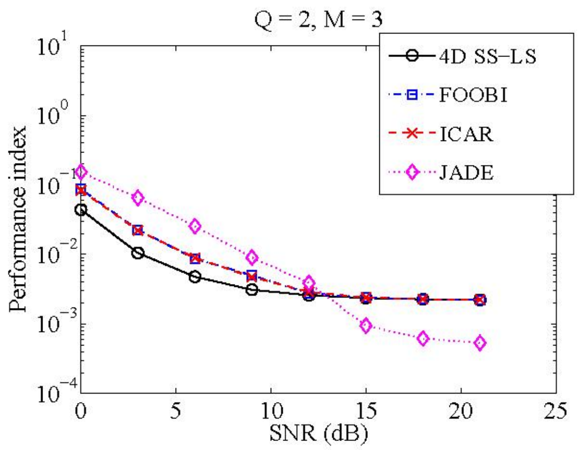

Fig. 3. Mean performance index vs. SNR: comparison with other BCI algorithms.

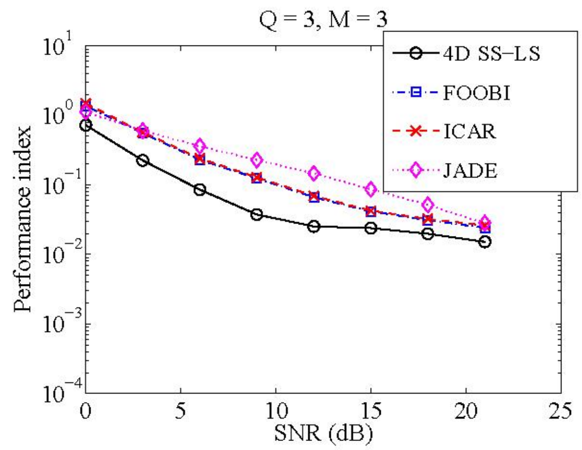

Fig. 4. Mean performance index vs. SNR: comparison with other BCI algorithms.

mixtures. The uniqueness issue has been addressed and simulation results have been provided showing that, in some specific scenarii, the SS-LS approach can perform better then existing identification algorithms. Some extensions of this work include the case of convolutive MIMO channels as well as the use of the SS-LS approach for the estimation of multipath parameters in wireless channels.

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